Optimal Margin Level in Futures Markets: Extreme Price Movements

FRANÇOIS M. LONGIN*

Along with price limits and capital requirements, the margin mechanism ensures the integrity of futures markets. Margin committees and brokers in futures markets face a trade-off when setting the margin level. A high level protects brokers against insolvent customers and thus reinforces market integrity, but it also increases the cost supported by investors and in the end makes the market less attractive.

This article develops a new method for setting the margin level in futures markets. It is based on "extreme value theory," which gives interesting results on the distribution of extreme values of a random process. This extreme value distribution is used to compute the mar-

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*Correspondence author, Department of Finance, ESSEC Graduate Business School, Avenue Bernard Hirsch, BP 105, 95021 Cergy-Pontoise, France.

■ *Francois Longin is an Associate Professor in the Finance Department at ESSEC in France and is affiliated with the CEPR.*

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gin level for a given probability value of margin violation desired by margin committees or brokers. Extreme movements are central to the margin-setting problem, because only a large price variation may cause brokers to incur losses. An empirical study using prices of silver futures contracts traded on COMEX is also presented. The comparison of the extreme value method with a method based on normality shows that using normality leads to dramatic underestimates of the margin level. Jrl Fut Mark 19: 127–152, 1999

INTRODUCTION

Along with price limits and capital requirements, the margin mechanism ensures the integrity of futures markets. The existence of margins decreases the likelihood of customers' default, brokers' bankruptcy and systemic instability of futures markets. Initial deposits and subsequent variation margin payments are designed to guarantee that investors will perform according to the terms of the contract. The risk of default, however, cannot be completely eliminated, because margin deposits cannot fully cover all adverse price changes. Default occurs when a trader reneges on the contract obligations. Such a situation arises when there is a large futures price change such that the investor's margin account is wiped out, the investor receives a margin call, but does not meet this margin call. Setting a high margin level thus reduces default risk. On the other hand, if the margin level is set too high, then the futures market will be less attractive for investors. Because maintaining funds on margin deposits amounts to a transaction cost on traders[1], an increase in margin requirements can be expected to decrease trading activity and thus brokers' commissions. And, as noted by Miller (1988), "driving major classes of users to seek alternatives to futures exchanges not only reduces the revenues of these exchanges but undermines the liquidity and market depth that is the very reason for their existence." It is in the self-interest of the exchanges to keep margins at appropriate levels: high enough to maintain market integrity yet low enough to maintain market liquidity.

Financial scholars have investigated how to set optimal margins by taking into account this trade-off. Telser (1981) and Hunter (1986) propose an economic model in which the margin level is endogenously determined. Figlewski (1984) and Gay et al. (1986) adopt a statistical ap-

[1]In some futures markets traders can post a portion of their margins in the form of Treasury bills or other liquid assets. As explained by Tomek (1985), this certainly reduces the cost associated with margins, although it could lead to nonoptimal portfolios and opportunity costs, because the bills cannot be used for other purposes.

proach: By taking a Brownian motion model for the dynamics of the rate of price change, they derive the probability of the first margin violation occurring on a given date for a given margin level. Tomek (1985), Edwards and Neftci (1988), and Warshawsky (1989) use the time series of actual movements in futures prices to compute margin exposure.[2]

The methods based on a parametric distribution assume that the (percentage) price change is normally distributed. However, there is now strong evidence that futures price changes are not normally distributed.[3] The tails of the empirical distribution of observed price changes appear to be thicker than the tails of the normal distribution, which means that large price changes actually occur more frequently than predicted by the unconditional normal model. As the integrity of the futures market is related to the occurrence of large price movements, the assumption of normality leads to underestimated margin levels due to the insignificant weight of the tails of the normal distribution. Kofman (1993) first recognized the importance of large price movements and suggested modeling the distribution tails explicitly for setting margins in futures markets. He proposed a nonparametric method using the so-called tail index, which reflects the weight of the distribution tails.

The sensitivity of the margin-setting problem to large price movements can be illustrated with the following model. A statistical distribution denoted by F is used to describe the behavior of futures price changes. This distribution can be either the historical distribution observed in the past or a given estimated distribution (say the normal distribution with estimated mean and variance). The margin level and the probability of margin violation are denoted respectively by ML and p. Because the margin level may be different for long and short positions, the two cases will be treated separately and different notations will be used: ML^{long} and p^{long} for a long position and ML^{short} and p^{short} for a short position. The case of a common margin level for both types of position ($ML = ML^{long} = ML^{short}$) will also be considered. The distribution F is used to relate the margin level to the probability of margin violation. A long position is affected by a large negative price movement. The probability of such an adverse movement, p^{long}, is given by eq. (1):

[2]These works assume that there is no feedback effect (in the sense that the margin level has no impact on the distribution of futures price changes, and especially not on volatility). This is consistent with the empirical literature on the margin-volatility relationship in futures markets. Fishe et al. (1990) and Moser (1992), among others, find no relationship between the margin level (or changes in the margin policy) and the volatility level (or subsequent changes in volatility).

[3]Cornew et al. (1984), for example, reject normality for a comprehensive selection of commodities and foreign exchange contracts. Using properly standardized price changes, Brorsen et al. (1993) still find evidence of nonnormality for agricultural commodities, livestock, metals and oil contracts as well as financial instruments.

$$p^{long} = \text{Prob}\ (\Delta P < -ML^{long}) = F(-ML^{long}), \tag{1}$$

where ΔP represents the change in the futures contract price (Δ represents the difference operator). Similarly, a short position is affected by a large positive price movement. The probability of such an adverse movement, p^{short}, is given by eq. (2):

$$p^{short} = \text{Prob}\ (\Delta P > ML^{short}) = 1 - F(ML^{short}). \tag{2}$$

As eqs. (1) and (2) show, the margin-setting problem is related to the occurrence of large futures price changes and thus to the tails of the distribution of futures price changes. Positions in futures contracts are not endangered by small or medium movements.

A major issue in the above model is the modeling of the distribution F. In practice, the empirical distribution is difficult to handle and a parametric model is often preferred. However, because there is no theoretical model for the exact distribution of price changes, an assumption has to be made. The method used in this article is based on "extreme value theory." This statistical theory gives interesting results for the distribution of extreme values of a random process. Extremes are precisely defined as the highest observation (the maximum) and the lowest observation (the minimum) over a given time period. Extreme value theory shows that the distribution for extremes observed over a *long* time period is largely independent of the parent distribution. In this article, the optimal margin level for a given probability value of margin violation is then computed using the distribution of extreme price changes. Focusing on the *extreme* price changes, rather than the price changes generally, allows avoidance of the difficult choice of a distribution for price changes.

A NEW METHOD TO SET MARGINS

The idea behind the method based on extreme price movements is first explained. Elements of extreme value theory and estimation procedures are then introduced. Finally, applications of extreme value theory to the setting of margins, price limits, and capital requirements are presented.

Basic Idea

There is neither an economic theory nor a statistical theory to assess the exact form of the distribution of futures price changes. Previous methods to set the margin level take a given form for this distribution, or estimate it empirically. The question asked is the following: What is the margin level associated with a given value of the probability of margin violation over one trading day? For a given value of the probability of margin violation or potential default tolerated by the margin committee, the answer is given by eqs. (1) and (2) for long and short positions. As the answer depends on the tails, the choice of the distribution F is critical. For example, a normal distribution would tend to underestimate the weight of the tails and thus to underestimate the margin level.

In this article the problem is put slightly differently. A period of n trading days is considered and the following question is asked: What is the margin level associated with the probability of daily margin violation over n trading days? The variable of interest has changed: The extreme price change observed over n trading days is the focus of interest, rather than the general price change. The probability of margin violation over n trading days, denoted by π, is thus different from the probability p associated with one trading day (or a longer grace period). For a long position the answer to the margin-setting problem is given by eq. (3):

$$\begin{aligned}\pi^{long} &= \text{Prob } (\text{Min}(\Delta P_1, \Delta P_2, \ldots, \Delta P_n) < -ML^{long}) \\ &= F_{\text{MIN}(n)}(-ML^{long}) \qquad (3)\end{aligned}$$

and for a short position by eq. (4):

$$\begin{aligned}\pi^{short} &= \text{Prob } (\text{Max}(\Delta P_1, \Delta P_2, \ldots, \Delta P_n) > ML^{short}) \\ &= 1 - F_{\text{MAX}(n)}(ML^{short}). \qquad (4)\end{aligned}$$

Here the distributions of interest, denoted as $F_{\text{MIN}(n)}$ and $F_{\text{MAX}(n)}$, are those of *extreme* price changes: the lowest daily price change (the minimum) and the highest daily price change (the maximum) observed over the n following trading days. The margin levels ML^{long} and ML^{short} are related to the probabilities of margin violation by extreme price changes, π^{long} and π^{short}. One could reason that the two approaches are equivalent and that nothing is gained by considering margin violation over n trading days instead of *one* trading day. Indeed, when the exact distribution F is known, this is the case: Nothing is gained as the two approaches give exactly the same result for the margin level.[4] But when the distribution

[4]When the distribution F is known, eqs. (6) and (7), given later in the text for an independent and identically distributed process, imply that the probability of margin violation by an extreme daily price change over n trading days, π, is directly related to the probability of margin violation by a daily price change, p, by: $\pi^{long} = 1 - (p^{long})^n$ for a long position and $\pi^{short} = 1 - (p^{short})^n$ for a short position, and that the usual approach is then equivalent to the extreme value approach.

F is not known, this is not the case. As the next subsection shows in detail, extreme value theory gives the form for the asymptotic distribution of the minimum and the maximum when selected over a long time-period. To compute the margin level, the unknown exact distribution of the extremes will be replaced in eqs. (3) and (4) by the asymptotic distribution given by extreme value theory, which can be known by estimation. One interesting feature of this theory is that the form of this distribution is largely independent of the process of daily price changes. Different processes of daily price changes lead to the *same* form of the distribution of extremes; the distributions of extremes derived from different processes are differentiated by the value of the parameters of the distributions of extremes only. The extreme value approach for the margin-setting problem exploits the generality of this result.

Because margin committees and brokers in futures markets must set the margin level (which is a component of trading costs) it is useful to have an explicit calculation of the margin level required to achieve a certain value of the probability of margin violation. Margin committees usually rely on statistical estimates to compute a margin level (Duffie 1989, p. 63). Although the distribution of futures price changes is important, other factors, such as liquidity, volume, open interest, concentration in futures positions, current and expected cash market conditions, and margins at other exchanges are taken into consideration [see Gay et al. (1986) and Rutz (1988)]. Let us consider two examples: the London Clearing House (LCH) and the Chicago Mercantile Exchange (CME).

The LCH undertakes clearing for the London International Financial Futures Exchange, the International Petroleum Exchange, the London Commodities Exchange and the London Metal Exchange. As explained by Vosper (1995), the setting of margin levels is a matter of judgment, and that judgment is formed by the overall context that affects the volatility of a given contract. This includes contracts' liquidity, and also political, economic, and market conditions. Even though the historical analysis of price data is essential, it is only one factor in a judgmental procedure. The LCH tends to rely less on normal-distribution-based calculations than on presentation of primary data on the recent past (typically the last three months). The analysis is also sensitive to large price movements, but the approach is nonmechanistic as such price movements may be overridden if unlikely to be repeated, or incorporated if they are likely to be repeated.

The CME has implemented the Standard Portfolio Analysis of Risk (SPAN) margining system, which provides a quantitative study of the risk of a position that is then used to compute the associated margin requirement (see Chicago Mercantile Exchange, 1994). As explained by Kupiec (1994), the SPAN system uses "what if" scenarios with price and volatility scan ranges to determine the margin requirement on portfolios that contain futures options contracts. The scan range is set equal to the margin requirement on a naked futures position and represents the maximum underlying price move in the scenario analysis. The margin committee usually considers the historical distribution of futures prices over the recent past and focuses on the 95% or 99% quantile of the distribution.

Theory of Extremes

This subsection presents some exact and asymptotic statistical results pertaining to extremes.[5]

Exact Results

Daily price changes are measured by a random variable denoted by ΔP. The assumption of pure randomness for futures price changes finds support in the theoretical literature (see Bachelier, 1900, and Samuelson, 1965, for a proof). Let us call f and F the probability density and cumulative distribution functions of the parent random variable ΔP, which can take values in the interval $[l, u]$. For example, a random variable distributed as the normal gives $l = -\infty$ and $u = +\infty$. Let $\Delta P_1, \Delta P_2, \ldots, \Delta P_n$ be n random price changes observed on days $1, 2, \ldots, n$. Extremes can be defined as minima and maxima of the n random variables ΔP_1, $\Delta P_2, \ldots, \Delta P_n$: $\mathrm{MIN}(n)$ represents the lowest daily price change (the minimum) and $\mathrm{MAX}(n)$ the highest daily price change (the maximum) observed over n trading days. As shown in Gumbel (1958), if the variables $\Delta P_1, \Delta P_2, \ldots, \Delta P_n$ are statistically independent and drawn from the same distribution (hypotheses of the random walk for futures prices), then the exact distribution of $\mathrm{MIN}(n)$ is given by eq. (5):

$$F_{\mathrm{MIN}(n)}(x) = 1 - [1 - F(x)]^n \tag{5}$$

and the exact distribution of $\mathrm{MAX}(n)$ by eq. (6):

$$F_{\mathrm{MAX}(n)}(x) = [F(x)]^n. \tag{6}$$

The distributions $F_{\mathrm{MIN}(n)}$ and $F_{\mathrm{MAX}(n)}$ depend mainly on the properties

[5]Gumbel (1958) gives a detailed, statistical exposition of extreme value theory and presents its applications in engineering to study strengths of materials, floods, droughts, air pollution, rainfalls, wind speed, and so on. Application in finance and insurance can be found in Embrechts et al. (1997). Advanced probability results can be found in Galambos (1978) and Leadbetter et al. (1983).

of F for large negative and large positive values of x. Indeed, for small absolute values of x, the influence of $F(x)$ decreases rapidly with n. Hence, the most important information about the extremes is contained in the tails of the distribution. From eq. (5), the limiting distribution of the extremes MIN(n) obtained by letting n tend to infinity is null for x less than the lower bound l and equal to one for x greater than l. From eq. (6), the limiting distribution of the extremes MAX(n) obtained by letting n tend to infinity is null for x less than the upper bound u and equal to one for x greater than u. In other words the limiting distributions are degenerate.

The exact results for the distribution of extremes are not, however, especially interesting. In practice, the distribution of the parent variable is not precisely known and, therefore, if this distribution is not known, neither is the exact distribution of the extremes. For this reason, the asymptotic behavior of the minimum MIN(n) and of the maximum MAX(n) is studied.

A Limiting Result: The Extreme Value Theorem

Tiago de Oliveira (1973) argues: "As, in general, we deal with sufficiently large samples, it is natural and in general sufficient for practical uses to find limiting distributions for the maximum or the minimum conveniently reduced and use them." To find a limiting distribution of interest, the random variable MIN(n) is transformed such that the limiting distribution of the new variable is a nondegenerate one. The simplest transformation is the standardization operation. The variate MIN(n) is adjusted with a location parameter β_n^{min} and a scale parameter α_n^{min} (assumed to be positive). Assuming the existence of a sequence of such coefficients ($\alpha_n^{min} > 0$, β_n^{min}), Gnedenko (1943) obtains three types of limiting distributions for the standardized extremes, whereas Jenkinson (1955) proposes a generalized equation.[6]

The limiting distribution of the standardized variable $(\mathrm{MIN}(n) - \beta_n^{min})/\alpha_n^{min}$ denoted as F_{MIN} is given by formula (7):

$$F_{\mathrm{MIN}}(x) = 1 - \exp[-(1 + \tau^{\min} \cdot x)^{1/\tau^{\min}}], \tag{7}$$

for $x < -1/\tau^{\min}$ if $\tau^{\min} < 0$ and for $x > -1/\tau^{\min}$ if $\tau^{\min} > 0$. A similar limiting distribution for the standardized maximum $(\mathrm{MAX}(n) - \beta_n^{max})/\alpha_n^{max}$ is given by formula (8):

[6]There are also results about the distribution of the second extreme, the third extreme and more generally the m^{th} extreme (Gumbel 1958, pp. 187–200).

$$F_{MAX}(x) = \exp[-(1 - \tau^{\max}\cdot x)^{1/\tau^{\max}}], \qquad (8)$$

for $x > 1/\tau^{\max}$ if $\tau^{\max} < 0$ and for $x < 1/\tau^{\max}$ if $\tau^{\max} > 0$. The parameter τ, called the tail index, determines the type of distribution: the limiting case ($\tau = 0$) corresponds to the double exponential Gumbel distribution ($[1 \pm \tau\cdot x]^{1/\tau}$ being interpreted as $e^{\pm x}$), $\tau < 0$ corresponds to the Fréchet distribution, and $\tau > 0$ to the Weibull distribution. The Gumbel distribution can be regarded as a transitional limiting form between the Fréchet and the Weibull distributions. The tail index reflects the fatness of the distribution (that is, the weight of the tails), whereas the parameters of scale α_n and of location β_n represent the dispersion and the average of the extremes, respectively. The extreme value theorem gives an interesting result: whatever the distribution of the parent variable ΔP, the limiting distribution of the extremes always has the same form. The distribution of the extremes for two different parent processes is differentiated by the values of the standardizing coefficients α_n and β_n and the tail index τ.

Distributions with exponentially decreasing tails, like the normal, provide a Gumbel extreme value distribution; fat-tailed distributions like Student-t, Cauchy, or other stable Paretien laws, lead to the Fréchet case whereas extremes obtained from bounded variables can be distributed either as a Weibull or a Gumbel distribution (see Gumbel, 1958, Chapter 4, and Galambos, 1978, Chapter 2, for details). Figure 1A illustrates the three types of extreme value distribution. Figure 1B gives the detail of the distribution tails: The Fréchet distribution is fat-tailed as its tail is slowly decreasing; the Gumbel distribution is thin-tailed as its tail is rapidly decreasing; and the Weibull has no tail—after a certain point there are no extremes.

The result of the extreme value theorem is found even if the basic assumption of an independent and identically distributed process is relaxed. Berman (1964) shows that the same result stands if normal variables are correlated and if the series of the squared correlation coefficients is finite. Extreme values are also influenced by the time-varying behavior of the second moment of the distribution. As shown in Longin (1997), the occurrence of extremes is linked to the persistence of shocks in volatility. De Haan et al. (1989) show that, if ΔP follows an ARCH process, then the variables MIN(n) and MAX(n) have a Fréchet limiting distribution. Leadbetter et al. (1983, Chapter 3) show that variations of the normal like an auto-regressive process, a discrete mixture of normal variables, and a mixed diffusion jump process with bounded jumps all lead to a Gumbel extreme value distribution.

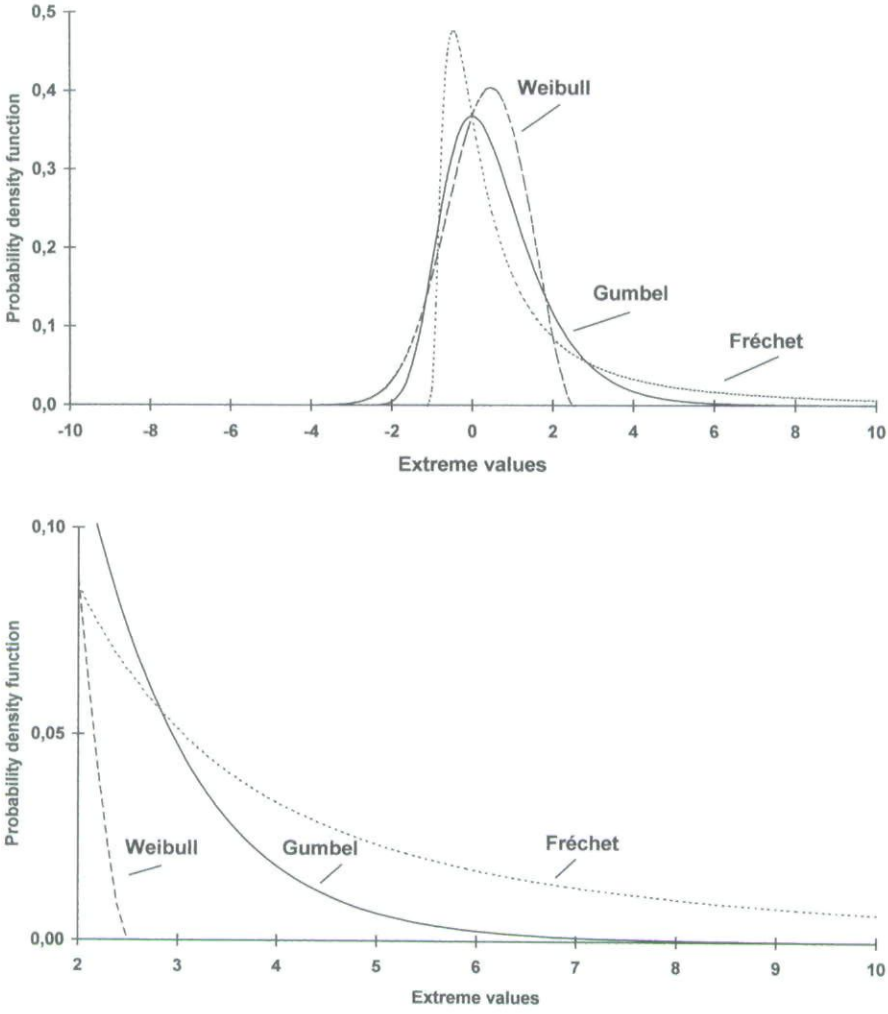

FIGURE 1

The Fréchet, Gumbel, and Weibull extreme value distributions. Figure 1A represents the extreme value distributions. According to the tail index value τ, three types of extreme value distribution can be distinguished: the Fréchet distribution ($\tau < 0$), the Gumbel distribution ($\tau = 0$), and the Weibull distribution ($\tau > 0$). The distribution for extreme price changes is a Fréchet if the distribution of price changes is fat-tailed, a Gumbel if the distribution of price changes is thin-tailed, and a Weibull distribution if the distribution of price changes has no tail (the price change and therefore the extreme price change are bounded). Figure 1B represents in detail the tails of the three types of extreme value distribution. The tail of the Fréchet distribution ($\tau < 0$) declines slowly at a power rate. The tail of the Gumbel distribution ($\tau = 0$) declines rapidly at an exponential rate. The Weibull distribution ($\tau > 0$) has no tail as there are no observations of price changes (nor therefore extreme price changes) beyond a certain point. The distributions represented in both figures are standardized extreme value distributions ($\alpha_n = 1$, $\beta_n = 0$) with tail index values equal to -0.8 for the Fréchet case, 0 for the Gumbel case and 0.4 for the Weibull case.

These results show that the basic assumptions of independence and identity of distribution are less important for extreme values than it would seem at first sight. However, in practice it is not necessary to know the process for daily price changes to investigate the behavior of extreme price changes, as in the estimation procedure the data speak for themselves.

Estimation Procedure

Turning to the statistical estimation, the asymptotic distribution of extremes contains three parameters: τ, α_n and β_n.[7] The first step consists of selecting the extremes from the data. Every day, a realization of the variable ΔP is observed. After n days, one thus gets n observations ΔP_1, $\Delta P_2, \ldots, \Delta P_n$, from which the lowest observation, denoted MIN_1, is extracted. From the next n observations $\Delta P_{n+1}, \Delta P_{n+2}, \ldots, \Delta P_{2n}$, another observation of the minimum called MIN_2 is extracted. If the database contains N^{obs} observations, then one gets N observations of minima $\mathrm{MIN}_1, \mathrm{MIN}_2, \ldots, \mathrm{MIN}_N$, where the variable N stands for the number of extreme observations.[8] The procedure of the selection of extremes is illustrated in Figure 2. Secondly, a regression method is used to get estimates of the parameters (see Gumbel, 1958, pp. 226, 260, and 296)). The sequence $\mathrm{MIN}_1, \mathrm{MIN}_2, \ldots, \mathrm{MIN}_N$ is arranged in increasing order to get an order statistic $\mathrm{MIN}'_1, \mathrm{MIN}'_2, \ldots, \mathrm{MIN}'_N$, which satisfies: $\mathrm{MIN}'_1 \leq \mathrm{MIN}'_2 \leq \ldots \leq \mathrm{MIN}'_{N-1} \leq \mathrm{MIN}'_N$. For each m ranging from 1 to N, the frequency $F_{\mathrm{MIN}(n)}(\mathrm{MIN}'_m)$ is a random variable lying between zero and one and with a mean equal to $m/(N+1)$. The idea behind the regression method is to equate the observed frequency (using the asymptotic distribution) to its theoretical mean as shown by eq. (9):

$$E\langle F_{MIN}(MIN'_m)\rangle$$
$$= E\left(1 - \exp\left[-\left(1 + \tau^{\min} \cdot \frac{MIN'_m - \beta_n^{\min}}{\alpha_n^{\min}}\right)^{1/\tau^{\min}}\right]\right) = \frac{m}{N+1}. \quad (9)$$

Twice taking the logarithm of the observed frequency $F_{\mathrm{MIN}(n)}(\mathrm{MIN}'_m)$ and of the theoretical mean $m/(N+1)$ and by adding an error term $\varepsilon^{\min}$, leads to a reduced form (10), which can be empirically estimated:

[7]Details and presentation of other methods can be found in Longin (1996).

[8]For a database containing N^{obs} daily observations and for a selection period of extremes containing n days, the number of extremes N is then equal to the integer part of N^{obs}/n.

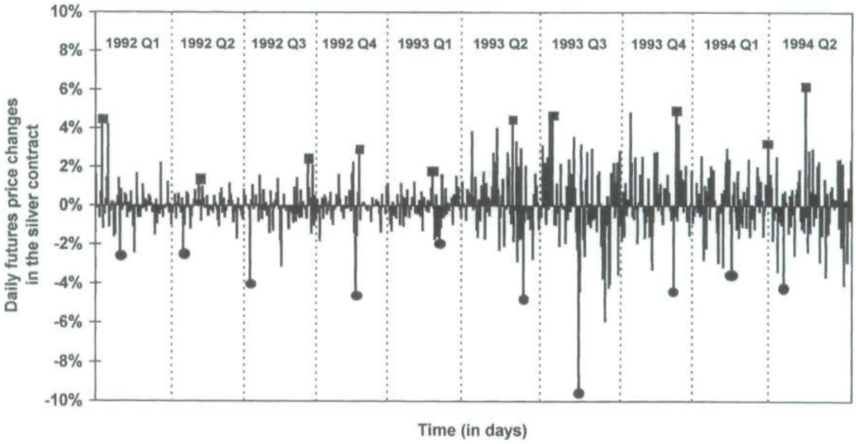

FIGURE 2

Selection of extreme price changes. This figure plots the history of the daily futures price changes in the silver contract traded on COMEX over the period January 1992 to June 1994 containing around 600 observations. Minimal price changes (marked by a circle) and maximal price changes (marked by a square) are selected over nonoverlapping quarters Q1, Q2, Q3, and Q4 every year. The example corresponds to the following parameters' values: $n = 60$ and $N = 10$; From the 600 observations of daily price changes, 10 observations of extreme price changes are obtained ($N = 10$). Extreme value theory is mainly concerned with the statistical properties of the extreme observations of the random process.

$$-Log\left[-Log\left(\frac{N+1-m}{N+1}\right)\right] = \frac{-1}{\tau^{\min}} \cdot Log\alpha_n^{\min} + \frac{1}{\tau^{\min}} \cdot Log(\alpha_n^{\min} + \tau^{\min} \cdot (MIN'_m - \beta_n^{\min})) + \varepsilon_m^{\min}. \quad (10)$$

As the Gumbel case is a limiting case ($\tau = 0$), another regression (11) is run for this distribution:

$$-Log\left[-Log\left(\frac{N+1-m}{N+1}\right)\right] = \frac{MIN'_m - \beta_n^{\min}}{\alpha_n^{\min}} + \eta_m^{\min}. \quad (11)$$

The maximum likelihood estimator is also used; it gives unbiased and asymptotically normal estimates. The equations are given in Tiago de Oliveira (1973). The system of nonlinear equations can be solved numerically using the Newton-Raphson iterative method; regression estimates are used as initial values of the algorithm.

Application to Futures Markets: Margins, Price Limits and Capital Requirements

Although this article focuses on the margin-setting problem, the margin system in futures markets is only one of the mechanisms to ensure market integrity. Two other important mechanisms are price limits and capital requirements for financial institutions.

Margins

The extreme value distribution is now used to derive the margin level *ML* for a given value of the probability of margin violation by an extreme price change, π. For a long position the two variables are related by eq. (12):

$$\pi^{\text{long}} = 1 - \exp\left[-\left(1 + \tau^{\min}\cdot\left(\frac{-ML^{long} - \beta_n^{\min}}{\alpha_n^{\min}}\right)\right)^{1/\tau^{\min}}\right] \quad (12)$$

and for a short position by eq. (13):

$$\pi^{short} = 1 - \exp\left[-\left(1 - \tau^{\max}\cdot\left(\frac{\text{ML}^{short} - \beta_n^{\max}}{\alpha_n^{\max}}\right)\right)^{1/\tau^{\max}}\right]. \quad (13)$$

Price Limits

It is sometimes argued that margin deposits should be accompanied by price limits. Edwards (1983) and Kyle (1988) list the advantages and drawbacks of this regulation. Among benefits most commonly cited, "price limits enable traders to better meet variation margin calls by giving them time to raise funds, and by making more predictable the amount of cash they may need during any given period of time. Limits also give the Clearing Association time to collect member margins, and FCMs time to collect customer margins." Daily price limits, however, hamper price discovery because the market price is prevented for some time from adjusting to its equilibrium level.

The likelihood of default due to margin violation is a decreasing function of the amount of margin deposits and an increasing function of the price limit. A safe system should place the price limit higher than the margin deposit—so that the margin deposit covers both the price change and a possible further adverse movement of the fundamental value (unknown because the market is closed).

Brennan (1986) develops a theory of price limits that explains both why the limit is set on a daily basis and why it is based on the price change from the close of the previous day. Under the assumption of either uniformity or normality for price changes, he derives an equilibrium for the price limit. In contrast, a statistical approach that does not rely on any statistical assumptions about the distribution of price changes may be used. The method presented above for margins could be similarly implemented to set optimal price limits up and down for a given probability for market prices to be limited up or limited down. The probability should reflect the trade-off between small and large price limits. As written by Kahl et al. (1985): "exchanges want to set daily price limits wide enough so that they are rarely reached. Then, daily price limits will rarely impede price discovery. However, exchanges want to impose daily price limits to limit the maximum daily loss."

However, as pointed out by a reviewer of this article, the existence of price limits may make the estimation of the extreme value distribution difficult, because the distribution of price changes may be altered once price limits are imposed. The mechanism of price limits may especially have an impact on the distribution tails as it eliminates the largest price movements (truncation effect). One way to deal with this problem may be to consider the price change at a lower frequency—for example, a week instead of a day. Considering the price change over a longer time-period may limit the impact of price limits on the observed distribution of price changes. Over a long time-period, the difference between the market price and the fundamental value will be smaller because the market has more time to incorporate information into prices.[9]

Capital Requirements

The integrity of financial markets is also enhanced by imposing adequate capital requirements on financial institutions. As explained in Dimson and Marsh (1995), capital requirements are needed to cover the position risk arising from the exposure of securities firms to fluctuations in the value of their holdings. Regulators are mainly concerned with events that may cause the default of some financial institutions because of effects on the whole financial system. The negative externalities arising from the failure of a financial business have forced regulators to impose minimum capital requirements to control the size and the frequency of default, such that the systemic risk remains small. An efficient procedure to compute capital requirement should focus on the tails of the distribution of price changes, because the types of risk of concern to regulators (default and systemic risks) may only be realized by an extreme price change, such as a stock market crash (see Longin, 1999, for a VaR method based on extreme values).

[9]Related to this issue, there is an interesting property of the extreme value distribution: The tail index is stable under time-aggregation (Feller (1971)).

EMPIRICAL RESULTS

This section first displays the results of estimation of the distribution of extreme price changes. Optimal margin levels obtained with this distribution and a comparison with other methods are then presented.

Estimation of the Distribution of Extreme Price Changes

The empirical study uses futures prices of silver contracts[10] traded on COMEX for the period 3 January 1975 to 30 June 1994, containing 4,989 trading days ($N^{obs} = 4{,}989$). Three different methods are used to build the time series of futures prices:

1) Closing prices on the nearby contract (when the nearby contract expires, prices on the contract with the closest maturity are then considered to build the time series). At the rollover point, there is often a discrete jump in the price level because of the change in the contract maturity. Such a characteristic of this method may create the following problem: a jump on the rollover date may sometimes lead to an extreme for a technical reason and not for an economic reason.

2) Closing prices on the nearby contract (up to the beginning of the delivery month) and then closing prices of the next nearest contract.

3) A weighted average of contract prices with different maturities. This method was developed by Geiss (1995) and allows one to maintain a continuity of data.

In practice, three different maturities were chosen, and the rollover date at which the nearby maturity disappeared from the index was arbitrarily set one week before the expiration date. Although Ma et al. (1992) showed that typical statistical tests of series of futures prices could be quite sensitive to the choice of the rollover date and linking method, this issue was not relevant for the study of extremes in the sense that extreme observations selected from the time series built using methods (1) or (2) were rarely associated with price changes at the rollover date (only one case out of 83); moreover, the parameters of the distribution of extreme price changes estimated from the different series were not significantly different. In the empirical study, the first method is used because margin committees or brokers are concerned with actual price changes observed in futures markets.

[10]Previous works have considered other markets: commodity markets by Kofman (1993), and equity markets by Longin (1995), Dewachter, and Gielens (1996) and Booth et al. (1997).

Following Figlewski (1984) and Kofman (1993), a time series of percentage logarithmic price changes is computed. The price change ΔP is defined by $\Delta P_t = 100 \cdot \mathrm{Log}[P_t/P_{t-1}]$ where P_t is the closing price on day t of the nearby contract. This definition presents several advantages: It provides the econometrician with a stationary time series; it is independent of the unit of measurement; and it is stable under time-aggregation. Margin committees, however, tend to express the margin level in terms of "dollars per contract" and not in terms of percentage. The two approaches are related: if the current futures contract price is \$100, a dollar margin of \$10 for a short position corresponds to a percentage margin rate of 9.53% [$= 100 \cdot \mathrm{Log}(110/100)$], and a dollar margin of \$10 for a long position corresponds to a percentage margin rate of -10.53% [$= 100 \cdot \mathrm{Log}(90/100)$].

Over the entire period, the average logarithmic price change is about -0.031% with a daily volatility of 1.874. The distribution of price changes is slightly negatively skewed (-0.27) and leptokurtic (2.83), indicating a small asymmetry in the distribution of price changes and the presence of large observations. There is little serial correlation in the time series of price changes: the first-order autocorrelation coefficient is equal to 0.049 and it is neither economically nor statistically significant. However, the time series of price changes is not independent, for significant serial correlation is found in squared price changes with a first-order autocorrelation estimate of 0.250.

Extreme price changes are of two types: minimal price changes and maximal price changes. They are selected over nonoverlapping quarters containing on average 60 trading days ($n = 60$). Minimal price changes have a mean of -4.83% and range from -12.80% to -1.89%. Maximal price changes have a mean of $+4.29\%$ and range from $+1.57\%$ to $+10.97\%$.

Turning to the estimation of the asymptotic distributions of minimal and maximal price changes, from the 4,989 observations of the database, 83 observations of quarterly extremes for each type are finally obtained ($N = 83$). Estimates of the parameters of the distribution of minimal price changes given by the regression method are for the scale coefficient $\alpha_n^{min} = 1.623$ (0.037), the location coefficient $\beta_n^{min} = -3.784$ (0.029) and the tail index $\tau^{min} = -0.096$ (0.018) with the standard error in parentheses. As the value of the tail index is negative and significantly different from zero, the distribution of minimal price changes is a Fréchet distribution.[11] Estimates of the parameters for the distribution of maxi-

[11]The model for the Gumbel distribution corresponds to the constraint ($\tau^{min} = 0$). A likelihood ratio

mal price changes are: $\alpha_n^{max} = 1.367$ (0.027), $\beta_n^{max} = 3.474$ (0.024) and $\tau^{max} = -0.047$ (0.015). The maximum likelihood method gives similar parameter estimate values. For example, for minima, one obtains: $\alpha_n^{min} = 1.458$, $\beta_n^{min} = -3.748$ and $\tau^{min} = -0.156$, and the standard error of the tail index estimate is equal to 0.074, also indicating a Fréchet distribution.

Optimal Margin Level

The problem of margin-setting in futures markets is now addressed by considering the *base* case of a margin level for a speculative (unhedged) account with a grace period of one day.[12] Three different methods to compute the margin level are compared for a given probability of margin violation.

The first method (Figlewski (1984)) assumes that daily price changes are drawn from a normal distribution with mean μ ($= -0.031$) and standard deviation σ ($= 1.874$). Using eqs. (1) and (2) with F standing for a normal distribution, the optimal margin level for a long position is given by eq. (14):

$$p^{long} = \frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{-ML^{long}} e^{-(x-\mu)^2/2\sigma^2}\, dx \tag{14}$$

and for a short position by eq. (15):

$$p^{short} = \frac{1}{\sqrt{2\pi}\sigma} \int_{ML^{short}}^{+\infty} e^{-(x-\mu)^2/2\sigma^2}\, dx. \tag{15}$$

In the case of the normal distribution, the margin level cannot be expressed analytically as a function of the probability of margin violation. However, it can be computed numerically.

The second method uses the extreme value distribution. Regression estimates are used ($\alpha_n^{min} = 1.623$, $\beta_n^{min} = -3.784$ and $\tau^{min} = -0.096$; $\alpha_n^{max} = 1.367$, $\beta_n^{max} = 3.474$ and $\tau^{max} = -0.047$). In the case of the

test asymptotically distributed as a chi-square with one degree of freedom leads to the rejection of the Gumbel case in favor of the Fréchet case (the test value is equal to 16.56 with a p-value less than 0.001).

[12]As noted by Gay et al. (1986), because futures traders are required to mark-to-market daily, the relevant time interval is one day. However, as a broker can give a grace period to some of his or her customers, it may be worth considering extreme price changes over a longer period. Examining the behavior of extreme price movements with a higher frequency may also be interesting, because intraday margin calls are usual during volatile periods—see, for example, the CFTC report (Commodity Futures Trading Commission, 1987) on the crash of October 1987.

extreme value distribution, the margin level can be analytically related to the probability of margin violation. For a long position the optimal margin level is given by eq. (16):

$$ML^{long} = -\beta_n^{\min} + \frac{\alpha_n^{\min}}{\tau^{\min}} \cdot [1 - (-Log(1 - \pi^{long}))^{\tau^{min}}] \quad (16)$$

and for a short position by eq. (17):

$$ML^{short} = \beta_n^{\max} + \frac{\alpha_n^{\max}}{\tau^{\max}} \cdot [1 - (-\mathrm{Log}(1 - \pi^{short}))^{\tau^{max}}]. \quad (17)$$

Note that the definition of the probability of an adverse movement is different in the usual approach (using price changes) and in the extreme value approach (using extreme price changes). Eqs. (5) and (6) obtained for an i.i.d. process imply that the probability of margin violation by an extreme daily price change observed over n trading days, π, is directly related to the probability of margin violation by a daily price change, p, by: $\pi^{long} = 1 - (p^{long})^n$ for a long position and $\pi^{short} = 1 - (p^{short})^n$ for a short position.[13]

Eqs. (14) to (17) provide a link between the margin level ML and the probability of margin violation p or π. These theoretical relationships derived from statistical models can be compared with the historical relationships (observed frequencies) as done by Tomek (1985), Edwards and Neftci (1988), and Warshawsky (1989). The empirical probability of margin violation is equal to the number (#) of observed price changes exceeding the margin level divided by the total number of observed price changes over the entire time-period (N^{obs}). For a long position it is given by eq. (18):

$$p^{long} = \frac{\#\{\Delta P_t \ such\ that\ \Delta P_t < -ML^{long}\}}{N^{obs}} \quad (18)$$

and for a short position by eq. (19):

$$p^{short} = \frac{\#\{\Delta P_t \ such\ that\ \Delta P_t > ML^{short}\}}{N^{obs}}. \quad (19)$$

Empirical results are presented in Table I for speculative long and short positions in the silver futures contract traded on COMEX. Values for the

[13]These equations are derived under the assumption of independence and identity of the distribution of price changes. In the general case, a parameter called the *extremal index* is introduced to take into account the dependence in the process of price changes (see Embrechts et al., 1997, Chapter 8).

TABLE I
Optimal Margin Level

	Margin level for a long position			*Margin level for a short position*			*Common margin level*
Probability of margin violation	*Normal*	*Extreme value*	*Historical*	*Normal*	*Extreme value*	*Historical*	*Extreme value*
0.50	4.29	4.30	4.30	4.23	3.98	4.30	4.10
0.25	4.89	5.75	6.28	4.83	5.23	5.32	5.47
0.10	5.50	7.68	8.48	5.44	6.72	6.51	7.17
0.05	5.91	9.29	8.84	5.85	7.83	7.77	8.49
0.01	6.75	13.56	n.a.	6.69	10.50	n.a.	11.80
0.005	7.09	15.76	n.a.	7.02	11.70	n.a.	13.37
0.001	7.81	21.88	n.a.	7.75	14.64	n.a.	17.40

Notes: this table gives the optimal margin level (as percentages) for a given probability of margin violation for speculative long and short positions in the silver futures contract traded on COMEX. Three distributions are used to compute the margin levels: the normal distribution for futures price changes (equations 14 and 15), the asymptotic distriubtion of extreme futures price chnages (equations 16 and 17), and the historical distribution of observed futures price chnages (equation 18 and 19). The results obtained by all three methods are computed with an extreme probability τ associated to extreme daily price changes observed over a quarter ($n = 60$). As explained in the text, the probability τ of margin violation by an extreme daily price change over a quarter is related to the probability p of margin violation by a daily price change. The last column gives the margin level common for both both long and short positions by the extreme value method (the distributions of minimal and maximal futures price changes are constrained to be symmetric). For low probability values, results are not available (n.a.) for the historical method because of the lack of data.

probability of margin violation π range from 0.50 to 0.001. For example, for a long position, with the extreme value distribution, the appropriate margin level should be 4.30% for $\pi^{long} = 0.50$ and 9.29% for $\pi^{long} = 0.05$. The remarkable result is that for conservative values of the probability of margin violation, the appropriate margin levels obtained under the assumption of normality are well below those obtained with the extreme value distribution. For example, for a long position, the appropriate margin level is equal to 5.91% under normality for $\pi^{long} = 0.50$ compared with 9.29% under the extreme value distribution. Similar conclusions apply to short positions. By using normality, the appropriate margin level is largely underestimated.

The extreme value distribution seems to fit well the behavior of extremes, because the margin levels given by the Fréchet distribution are similar to the level observed during the period. For $\pi^{long} = 0.05$, the level implied by the extreme value distribution, 9.29%, is close to the empirical level of 8.84%. For smaller probabilities, the scarcity of very large extremes does not allow a comparison—because results given by the nonparametric historical method are not available. As the extreme value method is parametric, it does not present such a disadvantage.

As the distribution of daily price changes is slightly skewed (reflected by more negative than positive extremes), some asymmetry is found in setting margins for long and short positions: For the same probability ($\pi = 0.05$), the margin level for a long position should be 9.29%, whereas for a short position it should be 7.83%. A higher level for long positions is needed to protect brokers from more numerous large price falls.

For practical purposes it is important to assess the stability of the results over time. The stability of optimal margin levels indeed reflects the degree of stationarity of the distribution of price changes itself. Longin (1995) investigates this issue and finds that the margin level required to cover price movements is sensitive to the time-period of analysis, as already noted by Warshawsky (1989).

Common Margin Level for Long and Short Positions

Until now the two tails (the negative and positive extreme price changes) have been treated separately. This led to different margin levels for long and short positions. This has theoretical appeal if the distribution of price changes is skewed, because the probability of a price increase of a given size need not equal the probability of a price decrease of the same size. Such an approach may, however, be inconsistent with the practice of setting margins by exchanges: No distinction is made between price increase and price decrease, and long and short investors face the same margin level. As pointed out to me by the Chicago Board of Trade, a plausible explanation is that "the simplicity of the rules (the costs of administrating them) outweighs any benefits of accounting for (likely small) skewness in the distribution." The London Clearing House also suggested that setting different margin levels for long and short positions may create inequality among market participants.

To respond to this practical constraint, a parametric extreme-based method is now proposed in order to give the same margin level for long and short positions. The parameters of the negative and positive extreme distributions are constrained to be the same: $\alpha_n = \alpha_n^{min} = \alpha_n^{max}$, $\beta_n = \beta_n^{min} = -\beta_n^{max}$ and $\tau = \tau^{min} = \tau^{max}$. Estimation procedures are modified to take account of these constraints. The regression method leads to the estimation of the model composed of two equations (one for the minimum and one for the maximum), such that all observations of extremes are used. As for the unconstrained case, the objective of the algorithm is to minimize the sum of the squared residuals of the model. Assuming the independence between ordered minima and maxima, the weighting matrix is taken to be diagonal. The bivariate model is defined by eqs. (20) and (21):

$$-Log\left[-\mathrm{Log}\left(\frac{N+1-m}{N+1}\right)\right] = -\frac{1}{\tau}\cdot \mathrm{Log}\alpha_n + \frac{1}{\tau}\cdot \mathrm{Log}[\alpha_n + \tau\cdot(MIN'_m - \beta_n)] + \varepsilon_m^{\min} \quad (20)$$

$$-\mathrm{Log}\left[-\mathrm{Log}\left(\frac{m}{N+1}\right)\right] = -\frac{1}{\tau}\cdot \mathrm{Log}\alpha_n + \frac{1}{\tau}\cdot \mathrm{Log}[\alpha_n - \tau\cdot(MAX'_m + \beta_n)] + \varepsilon_m^{\max}. \quad (21)$$

Regression estimates of the constrained model are now: $\alpha_n = 1.450$ (0.029), $\beta_n = 3.562$ (0.021) and $\tau = -0.089$ (0.013). Not surprisingly, the value of each parameter lies between the two values obtained for the unconstrained model, as reported above.

Optimal margin levels constrained to be equal for both long and short positions are reported in the last column of Table I. For a given value of the probability of margin violation, the level lies between the two levels of the unconstrained models. For example, for a probability value of margin violation of 5% ($\pi^{long} = 0.05$), the common margin level is equal to 8.49%, whereas the margin level for a long position is 9.29%, and the margin level for a short position is 7.83%.

Comparison with Current Margin Level on COMEX

Since the beginning of 1998, silver futures prices have been around \$6 per troy oz. For example, on 30 April 1998, the *Wall Street Journal* indicated a settlement price of \$6.192 for the May maturity. As a contract contains 5,000 troy oz, the price for one contract is indeed \$30,960. According to the NYMEX/COMEX web site, the margin level for speculators has changed a few times since the beginning of the year 1998: It increased from \$2,430 to \$3,105 on 4 February, then increased again to \$3,780 on the following day, and decreased to \$2,970 on 27 April. On 30 April 1998, the margin level (expressed as a percentage of the contract price) is around 9.59%. Using the estimated extreme value distribution, such a margin level corresponds to a value for the probability π around 0.05 for a long position. In other words, if no further action is taken by the Exchange in the future, the margin level will be exceeded with a probability of 5% over the next quarter. This probability level has to be appreciated by the Exchange itself, given its safety standard.

CONCLUSION AND FURTHER RESEARCH

This article develops a new approach for setting margins in futures markets. It is a parametric method, because it gives an analytical equation linking the margin level to the desired probability of margin violation. One important feature of the method is to take into account the occurrence of extreme price movements explicitly. These events are indeed at the very center of the margin-setting problem.

Further research using extreme value theory is now discussed. Investors often take positions in several markets, as noted by Edwards and Neftci (1988). If the extreme price variations in these markets are correlated, a method for an optimal margin level for a given probability of margin violation should take this correlation into account. An extension of this article could be to use the multivariate extreme value distribution by estimating the correlation between extreme price movements. This multivariate distribution could be used to assess the adequacy of margins; that is, to test if there is an equal probability of margin violation across markets, as done by Estrella (1988).

Margin committees and brokers usually set margin levels for hedge and spread positions which are different from the margin level for a speculative position. In a hedge position losses in the futures contract are offset with gains in the cash market while in a spread position, losses in a futures contract are offset with gains in a futures contract with a different maturity. However, hedges and spreads are not perfect as prices in the cash and futures markets and prices of futures contracts with different maturities are not perfectly correlated, especially during turbulent periods (the crash of October 1987 provides a good example of discrepancies in the different market segments). Because of hedge and spread risks, these positions are margined. Exchanges usually set these two margins via some simple, rather arbitrary rule, for example: seventy-five percent of the margin level of a speculative position. Further work would be to derive a margin level for these positions using the bivariate extreme value distribution to model the behavior of the different prices simultaneously.

Another line of research could be to derive optimal margin levels for options, using extreme value theory. As noted by Phillips and Tosini (1982), options and futures are closed substitutes, and disparity in margin requirements can create bias in trading. The computation of the probability of margin violation for the existing margin level of each market could tell which market provides the best protection against default, for brokers, or the least cost for investors.

Finally, more research should be done to assess the costs and benefits of margins: Is the cost of margins important when investors are allowed to deposit T-bills or other assets earning interest? How should this cost be measured? How should systemic risk be measured? Answers to these questions will help in the computation of a reliable probability of margin violation that can be used in statistical methods to derive an optimal margin level.

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