Is Bitcoin the new digital Gold?

Evidence from extreme price movements in financial markets*

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Abstract

Is Bitcoin the new digital Gold? To answer this question, we investigate the potential benefits of Bitcoin during extremely volatile periods. To this end, we use multivariate extreme value theory, which is the appropriate statistical approach to model the tail dependence structure. We focus on the extreme correlation. Considering first a position in equity markets, we find -similarly to previous studies- that the correlation of extreme returns increases during stock market crashes and decreases during stock market booms. Then, by combining each equity market with Bitcoin, we find that the correlation of extreme returns sharply decreases during both market booms and crashes, indicating that Bitcoin could play an important role in asset management. A similar result is obtained for Gold confirming its well-recognized status of a safe haven when a crisis happens. Furthermore, we find a low extreme correlation between Bitcoin and Gold, which implies that both assets can be used together in times of turbulence in financial markets. Such evidence indicates that Bitcoin can be considered as the new digital Gold. However, Gold can still play an important role in portfolio risk management.

Keywords: Bitcoin; diversification benefits; extreme correlation; extreme value theory; Gold; portfolio risk management; tail dependence

JEL classification: C46; F38; G01

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1. Introduction

Extreme adverse events in financial markets always represent a painful experience for investors. Thus, portfolio diversification during extremely volatile periods is of utmost importance for asset managers, financial advisors, and investors to control the risk level of their portfolio. A shift to secure assets during such periods is a strategy which is very frequently used to reduce portfolio riskiness. Over time, Gold has played the role of a safe haven;[2] the yellow metal has been considered as a suitable investment choice for portfolio diversification and portfolio hedging against adverse price movements (see Jaffe, 1989; Hillier et al., 2006; Baur and Lucey, 2010; Baur and McDermott, 2010; among others). Investors used to include Gold in their portfolios as it is characterized by high liquidity. It is also considered to be universal since it is globally accepted in transactions and provides thoughtful diversification benefits to traditional asset classes. Moreover, the purchasing power and the value of Gold have remained stable under the threat of erosion of the monetary or banking systems. Gold as a safe haven has over 5,000 years of history.

Over the past few years, Bitcoin has made a shattering entrance in the financial world. Bitcoin is an online communication protocol which uses a virtual currency, with the addition of electronic payments. Ten years after the seminal paper by Nakamoto (2008) introducing Bitcoin, the cryptocurrency has been a success in terms of popularity among both individual and institutional investors. Böhme et al. (2015) report in detail how Bitcoin works in their study, mentioning several potential innovative applications in several areas. Essentially Bitcoin has come out as something "*new*". Although it is now not the only cryptocurrency, Bitcoin is by far the largest in terms of market capitalization. Furthermore, the usefulness of Bitcoin has sparked interest for both academics and practitioners in the areas of statistics, risk management and asset management.

In this paper, we investigate the potential diversification benefits of Bitcoin during extremely volatile periods. To this end, we use extreme value theory, which is the appropriate approach to study this issue. In a multivariate framework, we focus on the extreme correlation, which summarizes the tail dependence structure of the return distribution. We develop a research strategy in *four* steps. First, we consider as a starting point a position in equity markets (Europe and the United States) and find that the extreme

[2] See Ranaldo and Soderlind (2010) for more information about safe haven assets.

correlation increases during stock market crashes and decreases during stock market booms. Such a stylized fact has been found in previous empirical studies (see Longin and Solnik, 2001; Ang and Chen; 2002, Hartmann et al., 2004; among others). Second, we combine each equity market with Bitcoin, and find that the correlation of extreme returns sharply decreases during both market booms and crashes, indicating that Bitcoin can play an important role in portfolio management during extremely volatile periods. Third, we combine each equity market with Gold, and find a similar result confirming the well-recognized status of Gold as a safe haven. Finally, we study the joint behavior of Bitcoin and Gold, and find a low extreme correlation, indicating that both assets can be useful together in times of turbulence in financial markets. Such evidence indicates that Bitcoin can be considered as the new digital Gold. However, Gold can still play an important role in portfolio risk management.

This paper is organized as follows: Section 2 details the research strategy followed in this study. Section 3 deals with the modeling of extremes. Section 4 details the estimation process, presents the testable hypotheses and reports the empirical results. Section 5 discusses the general economic backdrop of Bitcoin and Gold, compares the findings and assesses the joint potential of Bitcoin and Gold as diversifiers. Section 6 concludes by emphasizing the practical importance of our results in asset management.

2. Research strategy

This section presents our research strategy to investigate the potential diversification benefits of Bitcoin in asset management during extremely volatile periods. Our objective is to answer the following question: is Bitcoin the new digital Gold? To this end, we focus on extremely volatile periods, since such market conditions matter the most for investors. We use multivariate extreme value theory, which is the appropriate statistical approach to model the tail dependence structure. We focus on the extreme correlation. Our research strategy unfolds in four steps described below.

Step 1: Equity markets

We consider a position in equity markets (Europe and the United States). We focus on the correlation in equity markets during extremely volatile periods in order to assess diversification benefits in an equity position. Several empirical studies have found that the correlation of extreme returns increases during stock market crashes and decreases during stock market booms. Indeed, correlation is not related to market volatility per se, but to the market trend. This implies that the probability of large losses in two markets is significantly higher than the probability of large profits since downside market conditions constitute the driving force in equity correlation. The objective of this first step is to confirm this stylized fact about equity markets during extremely volatile periods.

Step 2: Equity markets and Bitcoin

We then combine each equity market with Bitcoin. The objective of this second step is to assess the potential diversification benefits of Bitcoin during extremely volatile periods. The usefulness of Bitcoin for investors would be characterized by a decreasing extreme correlation during market crashes implying diversification benefits. On the opposite, an increasing extreme correlation during market crashes would imply limited diversification benefits by including Bitcoin in an equity portfolio.

Step 3: Equity markets and Gold

We then combine each equity market with Gold. Several empirical studies have found a low correlation between equity markets and Gold during a financial crisis. The objective of this third step is to confirm the well-known status of Gold as a safe haven during stock market crashes. By looking at the extreme correlation between equity markets and Gold, we expect to find a decreasing extreme correlation confirming its well-recognized status of a safe haven when a crisis happens.

Step 4: Bitcoin and Gold

Finally, we consider a position in Bitcoin and Gold. The objective of this fourth step is to see if both assets can provide together diversification benefits during extremely volatile periods. The usefulness of both Bitcoin and Gold in an equity position would be characterized by a decreasing extreme correlation during market crashes implying extra diversification benefits. On the opposite, an increasing extreme correlation during market crashes would imply limited diversification benefits as Bitcoin and Gold are substitutable assets.

3. Modelling approach

This section describes the modelling approach for the behavior of extreme returns in financial markets. We model the bivariate tail dependence structure of the distribution of asset returns. We define extreme returns as return exceedances, that is, returns lower than a threshold for the left tail (negative return exceedances) and returns higher than a threshold for the right tail (positive return exceedances). First, we deal with the univariate modelling of extremes by fitting a general Pareto distribution (GPD) for each marginal distribution of return exceedances. To this end, we use the peaks-over-threshold method to select extreme returns for each distribution tail. Second, we deal with the bivariate modelling of extremes by fitting the Gumbel–Hougaard copula and focusing on the extreme correlation defined as the correlation of return exceedances.

3.1 Univariate modelling of extremes

Consider a sequence of independent and identically distributed random variables $\{X_1, X_2, \dots, X_n\}$ with a continuous cumulative distribution function F_X. For positive extremes, over a threshold $u > 0$, the distribution of exceedances $(X - u)$ denoted by F_X^u is given by:

$$F_X^u(x) = P(X - u \leq x | X > u) \; = \frac{F_X(u + x) - F_X(u)}{1 - F_X(u)}, 0 \leq x \leq x_{F_X} - u \qquad (1)$$

where $x = X - u$ is the exceedances and $x_{F_X} \leq \infty$ the right endpoint of F_X. The peaks-over-threshold method is an efficient method for modelling the extremes over a specific threshold under an unknown distribution (see Leadbetter, 1991).

For a large class of underlying distributions, Balkema and De Haan (1974) and Pickands (1975) showed that the excess distribution F_X^u can be approximated for large u by a GPD, $G_{\xi,\sigma}$, defined by:

$$G_{\xi,\sigma}(x) = 1 - p\left\{1 + \frac{\xi x}{\sigma}\right\}^{-1/\xi}, x > u \qquad (2)$$

where x represents the exceedances, p the tail probability of exceedances over threshold u, $\sigma > 0$ the scale parameter and $\xi \in \mathbb{R}$ the tail index. When $\xi > 0$, $G_{\xi,\sigma}$ corresponds to a heavy-tailed distribution (Fréchet type distribution). When $\xi \to 0$, $G_{\xi,\sigma}(x) \to 1 - exp\left(-\frac{x}{\sigma}\right)$ which is an exponentially declining tail distribution and corresponds to a thin-tailed distribution (Gumbel type distribution). When $\xi < 0$, $G_{\xi,\sigma}$ corresponds to a distribution with no tail or a finite distribution (Weibull type distribution).

For return distributions used in financial modelling, we can easily compute the parameters of the limit distribution. For example, the normal distribution leads to a GPD with $\xi = 0$. The Student-t distributions and stable Paretian laws lead to a GPD with $\xi > 0$. Furthermore, the GPD can be extended to processes based on the normal distribution: autocorrelated normal processes, discrete mixtures of normal distributions and mixed diffusion jump processes. They all have thin tails and their domain of attraction is a GPD with $\xi = 0$. De Haan et al. (1989) showed that if returns follow a GARCH process, then the extreme return has a GDP with $\xi > 0$.

3.2 Bivariate modelling of extremes

Consider a bidimensional vector of random variables denoted as $X = (X_1, X_2)$ with a bivariate distribution function F. Bivariate return exceedances correspond to the vector of univariate return exceedances defined with a bidimensional vector of thresholds $u = (u_1, u_2)$. The bivariate distribution can only converge toward a distribution characterized by a GPD for each margin and a dependence function. In this paper, we use copulas to model the dependence structure of vector X. Copulas are multivariate distributions with uniform [0,1] marginal distributions corresponding to transformed initial margins of distribution F (Sklar, 1959).

In a general form, a copula function under a common bivariate probability distribution F of vector Y of the transformed random variables $Y_i = F_{X_i}(X_i)$, for $i = 1,2$ is defined as:

$$C(u) = Pr\{Y_1 \leq u_1, Y_2 \leq u_2\} = F(F_{X_1}^{-1}(u_1), F_{X_2}^{-1}(u_2)) \tag{3}$$

The initial function F can arise from a copula function as $F(x) = C(F_{X_1}(x_1), F_{X_2}(x_2))$, which is an efficient transformation of F into C, and into univariate marginal distribution functions F_{X_i} (see Reiss and Thomas, 2001).

When dealing with extremes with heavy-tailed distributions and tail dependence, the appropriate statistical transformation for X is a standard Fréchet copula (Fréchet margins) in order to remove the influence of marginal aspects such that differences in distributions are due to dependence aspects (see Embrechts et al., 1999). Fréchet margins display dependency -either negative or positive- defined as Fréchet lower and upper bound copulas, which correspond to the limit cases of extreme dependency (see Yang et al., 2009). The Fréchet margins are given by $y_1 = -1/\log F_{X_1}(X_1)$ and $y_2 = -1/\log F_{X_2}(X_2)$ for X_1 and X_2, respectively, where F_{X_1} and F_{X_2} are the corresponding marginal distribution functions. Furthermore, $Pr(y_1 > u) = Pr(y_2 > u) \sim u^{-1}$ as $u \to \infty$. This corresponds to equally extreme events for each variable. The vector (Y_1, Y_2) is also described by the same dependence structure as in (X_1, X_2).

In this paper, we transform our data into unit Fréchet margins defined by the threshold u. Then, following Longin and Solnik (2001) and Poon et al. (2004), we consider two parametric dependence models as representatives of asymptotically independent and asymptotically dependent models.

As for the class of asymptotically independent models, the dependence function, denoted by D_G, is characterized by:

$$D_G(y_1, y_2) = \left(\frac{1}{y_1} + \frac{1}{y_2}\right) \tag{4}$$

where $y_i = -1/\log G_{\xi,\sigma}(x_i)$, for $i = 1,2$. The asymptotic independence of return exceedances is reached in many cases. When the components of the return distribution are independent, exact independence of extreme returns is obtained. However, asymptotic independence can arise even if the components of the return distribution are not independent. As proposed by Bortot et al. (2000), we employ the Gaussian model for Fréchet margins to model the asymptotically independent components, as follows:

$$D_G(y_1, y_2) = \Phi_2\left(\Phi^{-1}\left\{\exp\left(-\frac{1}{y_1}\right)\right\}, \Phi^{-1}\left\{\exp\left(-\frac{1}{y_2}\right)\right\}; \rho\right), \rho < 1 \tag{5}$$

where Φ_2 is a bivariate normal distribution with $\mu = (0,0)$ and $\Sigma = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}$.

As for the class of asymptotically dependent models, the dependence function D_G satisfies the following condition:

$$G(x_1, x_2) = exp\left(-D_G\left(-\frac{1}{y_1}, -\frac{1}{y_2}\right)\right), y_1, y_2 > 0 \tag{6}$$

We employ the logistic model proposed by the form of the dependence function of Gumbel-Hougaard copula (see Gumbel, 1960; 1961 and Hougaard, 1986) for Fréchet margins to model the asymptotically dependent components, as follows:

$$G_\alpha(x_1, x_2) = exp\left(-\left(y_1^{-1/\alpha} + y_2^{-1/\alpha}\right)^\alpha\right) \tag{7}$$

This model contains the special cases of asymptotic independence and total dependence. It is parsimonious as we only need one parameter to model the bivariate dependence structure of return exceedances: the dependence parameter α $(0 < \alpha \leq 1)$. The correlation coefficient of return exceedances ρ can be computed from the dependence parameter α of the logistic model by: $\rho = 1 - \alpha^2$. The special cases where α is equal to 1 and α is equal to 0 correspond to asymptotic independence, in which ρ is equal to 0, and total dependence, in which ρ is equal to 1, respectively (Tiago de Oliveira, 1973).

Although extreme value theory is based on large samples, in real applications, the limited number of return exceedances can lead to sample biases, especially as we move towards the distribution tails. In order to avoid such problems, we estimate a parametric bootstrap bias-corrected correlation coefficient for exceedances to reduce the estimation bias proposed by Gkillas and Longin (2018). To this end, we simulate from a bivariate extreme value distribution of a logistic type model following Stephenson (2003). By applying this procedure, we are able to avoid significant misleading results when the number of observations is limited.[3]

4. Empirical results

This section presents our empirical results. First, we discuss the data and data adjustments in order to work with stationary time-series. Second, we present the parameter estimates of the bivariate model for the tail dependence structure. Third, we provide some statistical tests related to normality and dependence based on the extreme correlation. Fourth, we discuss the main findings of our study.

4.1 Data and data adjustments

We analyze the tail dependence structure of international equity markets, Europe and the United States, vis-à-vis Bitcoin and Gold in a pairwise comparison. For the equity market in Europe (EU), we use the STOXX Europe 600 index, and for the equity market in the United States (US), we use the S&P 500 index. Both indices include the most heavily traded and liquid stocks with the largest market capitalization of their geographical zone.

Our empirical study covers the time-period from April 19, 2013 to April 17, 2018. Although Bitcoin started to be traded in 2010, we opt for the starting date of April 19,

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[3] The maximum likelihood procedure for fairly large samples allows the estimation of actual number of standard errors and confidence intervals (Coles et al., 2003). In such cases, the maximum likelihood procedure provides the most accurate estimates (Hosking and Wallis, 1987). Nevertheless, this approach is not applicable in some cases including the existence of biased error (Koch, 1991). On the one hand, if the sample size is quite large, the maximum likelihood estimator is a good choice (van Gelder et al., 1999). On the other hand, in the case of small samples, there are significant computational problems leading to unreliable estimates and sample bias (see Chaouche and Bacro, 2006; among others). Since that there are more adequate estimators, in this paper, we apply a parametric bias-corrected approach based on the maximum likelihood procedure to avoid such problems. Our approach reduces the sample bias observed is small sample taking into consideration the benefits coming from the standard maximum likelihood procedure.

2013 in order to avoid unreliable and spurious results due to the very low liquidity and resulting variability of Bitcoin during that period. From April 19, 2013, when Bitcoin prices broke for the first time the $100 threshold, the impact of liquidity on market prices became less important. In our study, we consider weekly returns so as to avoid the time lag bias between the equity markets in Europe and the United States. Data for Bitcoin, the STOXX Europe 600, the S&P 500 and Gold come from Bloomberg.

For each time-series of returns, we apply a data adjustment procedure based on the work of Gallant et al. (1992) to remove trends, and the work of MacNeil and Frey (2000) to take into account heteroskedasticity due to clusters. Thus, we limit the sample bias observed for serially-correlated and clustered data. We describe in detail our data adjustment procedure in Appendix 1.

4.2 Estimation of the parameters of the bivariate model

We discuss now the estimation of the parameters of the bivariate model for the tail dependence structure. Following our four-step research strategy, we present our empirical results in four sets of tables.

Table 1 refers to the bivariate tail dependence structure between the equity markets in Europe and the United States (EU/US). Table 2 refers to the bivariate tail dependence structure between each equity market and Bitcoin: Table 2A for Europe and Bitcoin (EU/BTC), and Table 2B for the United States and Bitcoin (US/BTC). Table 3 refers to the bivariate tail dependence structure between each equity market and Gold: Table 3A for Europe and Gold (EU/Gold), and Table 3B for the United States and Gold (US/Gold). Table 4 refers to the bivariate tail dependence structure between Bitcoin and Gold (BTC/Gold). Overall, we study the following pairs among international equity indices, Bitcoin and Gold, namely EU/US, EU/BTC, EU/Gold, US/BTC, US/Gold and BTC/Gold. For each table, Panel A refers to negative return exceedances in the left tail of the distribution, and Panel B to positive return exceedances in the right tail.

We provide maximum likelihood estimates of the parameters of the bivariate extreme distribution for both fixed and optimal thresholds. We define fixed threshold with tail probability levels across the entire range of the distribution of returns as: 50%, 40%, 30%, 20%, 10% and 5%. For each pair, we use the same value of probability level p for return exceedances in each time-series. We also compute optimal thresholds following the procedure described in Appendix 2. As explained by Jansen and de Vries (1991), optimal thresholds optimize the trade-off between inefficiency and sample bias. We report these estimates on the last line of each table panel. We report the following parameters: the threshold u associated with the tail probability p, the dispersion parameter σ, the tail index ξ for each series, the dependence parameter α of the logistic function used to model the dependence between extreme returns, and the extreme correlation ρ. We give the standard errors of the estimates in parentheses.

A graphical representation of our estimates in Tables 1-4 is also given in Figures 1-4, corresponding to each step of our research strategy. In these figures, we depict the evolution of the correlation of return exceedances moving towards the distribution tails. The value of the tail probability p is used to define return exceedances. These figures also graphically capture the potential asymmetry between negative and positive return exceedances in the left and right distribution tails. The solid line represents the correlation between actual return exceedances obtained from the estimation of the bivariate distribution modelled via the logistic function. The dotted line represents the theoretical correlation between simulated normal return exceedances, assuming a bivariate normal return distribution with parameters equal to the empirically-observed means and covariance matrix of returns.

4.3 Statistical tests related to normality and dependence

We provide statistical tests based on the extreme correlation to study the issues of normality and dependency. First, we test if the observed extreme correlation corresponds to the case of normality. Any statistical deviation from normality is important in practice as normality remains the standard assumption for modeling returns in asset management. Indeed, if the assumption of bivariate normality is violated, the use of normality could provide misleading results to describe portfolio risk under extreme market conditions, and misguided diversification strategies. Second, we test if the observed extreme correlation corresponds to the case of independence or the case of total dependence. A statistical deviation from independence implies that the diversification benefits are limited, and even wiped out in the case of total dependence. The last columns of each table panel report the Wald tests of these hypotheses with the *p*-values in brackets.

With respect to normality, we consider two cases: the asymptotic case and the finite-sample case. The former case considers the correlation of normal return exceedances of thresholds tending to infinity, denoted by ρ_{nor}^{asy}, which is theoretically equal to 0. The latter case considers the correlation of return exceedances over a given finite threshold u, denoted by $\rho_{nor}^{f.s.}(u)$.

- $H_0: \rho = 0$. We test the null hypothesis of *asymptotic* normality. That is if the observed extreme correlation is equal to the extreme correlation in the asymptotic case obtained with a normal distribution of returns, ρ_{nor}^{asy}, which is equal to 0.
- $H_0: \rho = \rho_{nor}^{f.s.}(u)$. We test the null hypothesis of normality in the *finite-sample* case. That is if the observed extreme correlation is equal to the extreme correlation in the finite-sample case obtained with a normal distribution of returns. In the finite-sample case, we compute $\rho_{nor}^{f.s.}(u)$ over a given finite threshold u by simulation assuming that returns follow a bivariate normal distribution with parameters equal to the empirically-observed means and covariance matrix of returns.

With respect to the issue of dependence, we consider the two limit cases: independence and total dependence. The former case corresponds to an extreme correlation, ρ_{ind}, which is equal to 0, and the latter to an extreme correlation, ρ_{dep}, which is equal to 1.

- $H_0: \rho = 0$. We test the null hypothesis of *asymptotic independence* of extremes. That is if the observed extreme correlation is equal to the extreme correlation obtained under asymptotic independence of extremes, ρ_{ind}, which is equal to 0.
- $H_0: \rho = 1$. We test the null hypothesis of *total dependence* of extremes. That is if the observed extreme correlation is equal to the extreme correlation obtained under total dependence of extremes, ρ_{dep}, which is equal to 1.

4.4 Main empirical results

We now present our main empirical results about the estimation of the parameters of the bivariate model for the tail dependence structure. We follow our four-step research strategy highlighting the major findings in each step.

Step 1: Equity markets

Table 1 refers to the bivariate tail dependence structure between the equity markets in Europe and the United States (EU/US). We confirm the stylized fact of the behavior of equity markets during extremely volatile periods. We find that the tail dependence increases in bear markets and decreases in bull markets. Longin and Solnik (2001) found similar results between the main European equity markets (France, Germany, the United Kingdom) and the US equity market. The level of extreme correlation during stock market crashes is even higher in our study which uses a more recent time-period: 0.878 *vs* 0.571 for the correlation for negative return exceedances (at optimal threshold levels). The level of extreme correlation during stock market booms is also higher in our study: 0.384 *vs* 0.140 for the correlation for positive return exceedances.

More specifically, for negative return exceedances (Panel A), we observe that the correlation of return exceedances ρ increases across the left tail of the distribution. It is equal to 0.888 for p = 50% and 0.890 for p = 5%. The correlation of return exceedances ρ at the optimal thresholds is equal to 0.878. With respect to asymptotic normality, we reject the null hypothesis: $H_0: \rho = \rho_{nor}^{asy}$, as the first Wald test reveals across the entire range of the left distribution tail. The value of this test is equal to 26.641 for p = 50% and 203.487 for p = 5%. At the optimal thresholds, it is equal to 63.419 and leads to a strong rejection of the null hypothesis too. With respect to normality in the finite-sample case, we also reject the null hypothesis $H_0: \rho = \rho_{nor}^{f.s.}(u)$ moving to the left endpoint of the distribution for tail probability levels lower than 20%, as the second Wald test suggests. The value of this test is equal to 0.694 for p = 50%, 2.152 for p = 20%, and 3.726 for p = 5%. At the optimal thresholds, it is equal to 3.130 and leads to a rejection of the null hypothesis too. With respect to the asymptotic independence of extremes, we reject the null hypothesis: $H_0: \rho = 0$, as the first Wald test reveals across the entire range of the left distribution tail. With respect to the total dependence of extremes, we reject the null hypothesis: $H_0: \rho = \rho_{dep}$ for all threshold values. The value of this test is equal to 3.256 for p = 50% and 25.008 for p = 5%. At the optimal thresholds, it is equal to 8.678 and leads to a strong rejection of the null hypothesis too.

As for positive return exceedances (Panel B), we observe that the correlation of return exceedances ρ declines across the right tail of the distribution. It is equal to 0.864 for p = 50% and 0.521 for p = 5%. The correlation of return exceedances ρ at the optimal thresholds is equal to 0.668. With respect to asymptotic normality, we reject the null hypothesis $H_0\colon \rho = \rho_{nor}^{asy}$, as the first Wald test reveals across the entire range of the right distribution tail. The value of this test is equal to 24.194 for p = 50% and 17.260 for p = 5%. At the optimal thresholds, it is equal to 61.206 and also leads to a strong rejection of the null hypothesis. Furthermore, with respect to normality in the finite-sample case, we cannot reject the null hypothesis $H_0\colon \rho = \rho_{nor}^{f.s.}(u)$ moving to the right endpoint of the condition distribution for all values of u under consideration, as the second Wald test suggests. The value of this test is equal to 0.169 for p = 50% and 0.548 for p = 5%. At the optimal thresholds, it is equal to 0.199. With respect to the asymptotic independence of extremes, we reject the null hypothesis: $H_0\colon \rho = 0$, as the first Wald test reveals across the entire range of the right distribution tail. With respect to the total dependence of extremes, we reject the null hypothesis $H_0\colon \rho = \rho_{dep}$ of total dependence in all cases. The value of this test is equal to 27.155 for p = 50% and 32.592 for p = 5%. At the optimal thresholds, it is equal to 90.987.

The asymmetry between negative and positive return exceedances is confirmed by Figure 1, which refers to the bivariate tail dependence structure between the European and United States return exceedances (EU/US). As shown in Figure 1, the correlation of negative return exceedances is always greater than the correlation of positive return exceedances. The difference is statistically significant at the 5% confidence level.

Step 2: Equity markets and Bitcoin

Tables 2A and 2B refer to the bivariate tail dependence structure between each equity market and Bitcoin: Europe and Bitcoin (EU/BTC) and the US and Bitcoin (US/BTC). In this step, we combine each equity market with Bitcoin to assess the potential diversification benefits of Bitcoin during extremely volatile periods. We find that the tail dependence between each equity market and Bitcoin decreases in both bear and bull markets. Thus, Bitcoin could provide significant diversification benefits to investors.

More specifically, as for Table 2A, we observe that the dependency declines moving towards the distribution tails. Regarding negative return exceedances (Panel A), the correlation of return exceedances is equal to 0.477 for p = 50% and 0.019 for p = 5%. Regarding positive return exceedances (Panel B), the correlation of return exceedances stands is equal to 0.609 for p = 50% and 0.084 for p = 5%. Furthermore, we accept the null hypothesis that the correlation of return exceedances follows a bivariate-normal distribution in most fixed thresholds in both distribution tails. As for Table 2B, a similar conclusion is obtained. The dependency declines moving towards the distributions tails and we accept the null hypothesis that the correlation of return exceedances follows a bivariate-normal distribution in most fixed thresholds in both distribution tails. Regarding negative return exceedances (Panel A), the correlation of return exceedances is equal to 0.547 for p = 50% and 0.123 for p = 5%. Regarding positive return exceedances (Panel B), the correlation of return exceedances is equal to 0.599 for p = 50% and 0.200 for p = 5%.

Figures 2A and 2B depict the bivariate tail dependence structure between each equity market and Bitcoin (EU/BTC and US/BTC). Unlike Figure 1 for equity markets alone, we observe that the extreme correlation for both negative and positive return exceedances decreases when we go further into the tails. Moreover, this statistical behavior appears to be symmetric.

Step 3: Equity markets and Gold

Tables 3A and 3B refer to the bivariate tail dependence structure between each equity market and Gold: Europe and Gold (EU/Gold) and the US and Gold (US/Gold). In this step, we combine each equity market with Gold to confirm its well-known diversification benefits during extremely volatile periods. We find that the tail dependence between each equity market and Gold decreases in bear markets. Thus, it confirms the status of Gold as a safe haven.

More specifically, as for Table 3A, we observe that the dependency declines moving towards the distribution tails. We observe the same bivariate patterns among the dependency of negative returns exceedances between the pairs, the one of which is Bitcoin or Gold. In positive returns exceedances, the corresponding dependency is always greater in the pairs where Gold exists. The correlation of return exceedances is equal to 0.522 for p = 50% and 0.060 for p = 5% for negative return exceedances (Panel A). Regarding positive return exceedances (Panel B), the correlation of return exceedances is equal to 0.606 for p = 50% and 0.372 for p = 5%. As for Tables 3B, a similar conclusion is obtained. The dependency declines moving towards the distributions tails. We do not reject the null hypothesis that the correlation of return exceedances follows a bivariate normal distribution for most fixed thresholds in both distribution tails. Regarding negative return exceedances (Panel A), the correlation of return exceedances is equal to 0.558 for p = 50% and 0.089 for p = 5%. Regarding positive return exceedances (Panel B), the correlation of return exceedances is equal to 0.614 for p = 50% and 0.259 for p = 5%.

Figures 3A and 3B depict the bivariate tail dependence structure between each equity market and Gold (EU/Gold and US/Gold). Unlike Figure 1 for equity markets alone, we observe that the extreme correlation for both negative and positive return exceedances decreases when we go further into the tails. As for Bitcoin, this statistical behavior appears to be symmetric.

Step 4: Bitcoin and Gold

Table 4 refers to the bivariate tail dependence structure between Bitcoin and Gold (BTC/Gold). In this step, we consider a position in Bitcoin and Gold only to see if both assets can provide diversification benefits during extremely volatile periods at the same time. We find that the tail dependence between Bitcoin and Gold decreases in both bear and bull markets. Thus, it indicates that both Bitcoin and Gold can be used together in times of turbulence of financial markets.

As for Table 4, the dependency also declines moving towards the distribution tails. We do not reject the null hypothesis that the correlation of return exceedances follows a bivariate normal distribution for most fixed thresholds in both distribution tails. More specifically, the correlation of negative return exceedances (Panel A) is equal to 0.520 for p = 50% and 0.083 for p = 5%. The correlation of positive return exceedances (Panel B) is equal to 0.590 for p = 50% and 0.106 for p = 5%.

Figure 4 depicts the bivariate tail dependence structure between Bitcoin and Gold (BTC/Gold). Unlike Figure 1 for equity markets, we observe that the extreme correlation for both negative and positive return exceedances decreases when we go further into the tails.

5. Bitcoin vs Gold

In this section, we evaluate the potential diversification benefits of both Bitcoin and Gold during extremely volatile periods in equity markets. We first discuss the general economic backdrop of Bitcoin and Gold. We then compare the extreme correlation between equity markets, and Bitcoin or Gold. Finally, we assess the joint potential of Bitcoin and Gold as diversifiers for equity positions.

5.1 Economic backdrop

Bitcoin and Gold are two fundamentally different assets in several respects. On the one hand, Bitcoin exhibits a very shadowy background, including accusations of securities theft, fraud, and criminal activity (see Gandal et al., 2018; among others). It is also free of sovereign risk since it is independent from regulatory authorities, central banks and governments.[4] On the other hand, Gold exhibits a very good reputation. It is universal since it is globally accepted in transactions. Bitcoin and Gold also present similarities. Mainly, they are both non-productive assets and speculative investments as they do not produce future cash flows. Gold is inscribed in the memory of investors as a safe haven during various economic disasters. Recently, the financial press has debated if Bitcoin could present the same capabilities as Gold, while the academic literature cannot provide convincing answers on this topic.[5]

To contribute to the current comparison between Bitcoin and Gold, we study if Bitcoin has an advantage over Gold in terms of diversification benefits during downside market conditions. Since such market conditions matter the most for investors, we base our contribution on multivariate extreme value theory, which constitutes the proper statistical tool to model the dependence structure during extremely volatile periods. We also wonder if both Bitcoin and Gold can be useful together as safe havens.

5.2 Diversifiers for equity markets: Bitcoin or Gold?

Table 5 compares the results obtained in Steps 2 and 3 of our research strategy. Panel A reports the extreme correlation between the European equity market and Bitcoin $\rho^{EU/BTC}$, and between the European equity market and Gold $\rho^{EU/Gold}$. Panel B reports the extreme correlation between the US equity market and Bitcoin $\rho^{US/BTC}$, and between the US equity market and Gold $\rho^{US/Gold}$. In each panel, we test the following null hypotheses: $H_0: \rho^{EU/BTC} = \rho^{EU/Gold}$ and $H_0: \rho^{US/BTC} = \rho^{US/Gold}$, by a Wald test, to assess the potential advantages of Bitcoin and Gold in terms of diversification benefits during downside market conditions.

As for the European equity market (Panel A), for negative return exceedances, we cannot reject the null hypothesis of equality $H_0: \rho^{EU/BTC} = \rho^{EU/Gold}$ at optimal threshold levels as the value of the Wald test is equal to 0.482. As for positive return exceedances,

[4] Since Bitcoin uses the blockchain technology which ensures that any transaction is unique, and users can complete transactions without any intervention from regulatory authorities, central banks and governments. See Yermack (2017) for additional information regarding Blockhains.

[5] This point has been highlighted by the financial press. See Mackintosh (2017), Price (2018), Somerset Webb (2018) and Taplin (2018) among others.

we also cannot reject the null hypothesis as the value of the Wald test is equal to 0.785. As for the US equity market (Panel B), for negative return exceedances, we cannot reject the null hypothesis of equality $H_0: \rho^{US/BTC} = \rho^{US/Gold}$ at optimal threshold levels, since the value of the Wald test is equal to 0.886. As for positive return exceedances, we cannot also reject the null hypothesis as the value of the Wald test is equal to 0.872.

Figure 5A depicts the extreme correlation between the European equity market and Bitcoin, and between the European equity market and Gold. Figure 5B depicts the extreme correlation between the US equity market and Bitcoin, and between the US equity market and Gold. In these figures, the differences in the extreme correlation between the pairs under consideration are mostly statistically non-significant, with the exception of the positive return exceedances in the European equity market.

Overall, considering a separate addition of Bitcoin or Gold in an equity position, our findings show that an equity position including Gold does not have any significant advantage over an equity position including Bitcoin during extremely volatile periods. Although advantages and disadvantages can be found for both speculative assets, our extreme value analysis contributes to the debate on which asset is superior and why; our approach is not based on philosophical premises of progressivists and conservationists yet based on a rigorous statistical analysis for portfolio risk management. Consequently, from the perspective of diversification benefits, we conclude that Bitcoin can be considered as the new digital Gold.

5.3 Joint diversifiers for equity markets: Bitcoin and Gold?

The findings obtained in Step 4 of our research strategy reveal clear evidence that both Bitcoin and Gold can be useful together in times of turbulence in financial markets. We remind here that we find a decreasing correlation by going further into both the left and right tails. We observe very low correlation levels: the correlation of negative return exceedances is equal to 0.054 at optimal threshold levels, and the correlation of positive return exceedances is equal to 0.024. Both assets, Bitcoin and Gold, could then be added to a position in equity markets to provide extra diversification benefits.

From a portfolio management point of view, for the question "Bitcoin *vs* Gold", our empirical analysis shows that both Bitcoin *and* Gold is the best answer to diversify a position in equity markets during extremely volatile periods.

6. Conclusion

In this paper, we use multivariate extreme value theory, which is the proper statistical approach to deal with extremes, to investigate the potential diversification benefits of Bitcoin during extremely volatile periods. We focus on the correlation of return exceedances analyzing the tail dependence structure of international equity markets, Europe and the United States, vis-à-vis Bitcoin and Gold in a pairwise comparison.

We proceed in our analysis developing a research strategy in four steps. In Step 1, we consider a position in equity markets, and find -similarly to previous studies- that the correlation of extreme stock returns increases during stock market crashes and decreases during stock market booms. In Step 2, we combine each equity position with Bitcoin, and find that the correlation of extreme returns sharply decreases during both market booms and crashes, indicating that Bitcoin can play an important role in asset management during extremely volatile periods to provide diversification benefits. In Step 3, we similarly combine each equity position with Gold, and obtain a similar result, confirming its well-recognized status of safe haven in asset management when a crisis happens. In Step 4, we combine Bitcoin and Gold, and find a low correlation of return exceedances, indicating that both assets can be useful together in times of turbulence in financial markets. Such evidence shows that Bitcoin can be considered as the new digital Gold, yet Gold can still play an important role in portfolio risk management.

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Appendix 1. Procedure to obtain stationary return series

In order to apply extreme value theory, it is important to work with stationary time-series. To deal with this issue we used the 3-step procedure developed by Gallant et al. (1992) reproduced below.

Step 1

First, we de-trend the mean by regressing the raw original series on a set of explanatory variables that take into account the time trends (linear and quadratic) and several seasonality effects, as follows:

$$\boldsymbol{r} = \boldsymbol{x} \cdot \boldsymbol{\beta} + \boldsymbol{u} \qquad \text{A1.1}$$

with $\boldsymbol{r}$ being log-returns. The matrix $\boldsymbol{x}$ comprises the following regressors: a constant term, a dummy variable for each day of the week, except Monday to avoid multicollinearity and without considering Saturdays and Sundays; four dummy variables that refer each to one particular period in January, and that all together cover the 31 days in January (1-7, 8-14, 15-21, 22-31); four dummy variables that refer each to one particular period in December, and that all together cover the 31 days in December (1-7, 8-14, 15-21, 22-31); one dummy variable for each month of the year, except January, February and December to avoid multi-colinearity; two dummy variables to take into account time trends, one linear and one quadratic; four dummy variables, that define, respectively, situations in which there is a gap of 1 day, 2 days, 3 days or 4 days between two consecutive trading days. In total, $\boldsymbol{x}$ comprises 28 regressors, including the constant. The aforementioned regressors are meant to take into account the seasonality of a return series.

Step 2

Second, we de-trend the variance of the time-series by running the subsequent regression, as follows:

$$\mathbf{log}\, \boldsymbol{u}^{2} = \boldsymbol{x}'\boldsymbol{\gamma} + \boldsymbol{\epsilon} \qquad \text{A1.2}$$

where it has to be noticed that the same set of explanatory variables is used in order to remove the trend from the variance.

Step 3

Third, we perform the following transformation to compute the adjusted time series, as follows:

$$\boldsymbol{r_{adj}} = \boldsymbol{a} + \boldsymbol{b}\left(\frac{\widehat{\boldsymbol{u}}}{\frac{\boldsymbol{e^{x'\gamma}}}{\boldsymbol{2}}}\right) \qquad \text{A1.3}$$

where the coefficients $\boldsymbol{a}$ and $\boldsymbol{b}$ in (A1.3) are determined by solving a system of two equations with two unknowns, where the adjusted time series is required to have the same mean and variance of the original series.

Appendix 2. Derivation of the maximum likelihood function

To estimate the parameters of the model presented in Section 3, we use the maximum likelihood method developed by Ledford and Tawn (1997) and applied by Longin and Solnik (2001) reproduced below. This appendix presents the construction of the likelihood function in detail.

The method is based on a set of assumptions. Returns are assumed to be independent. The thresholds u_1 and u_2 used to select return exceedances (or equivalently the tail probabilities p_1 and p_2) are independent of the variables and time. The method is also based on a censoring assumption. For thresholds u_1 and u_2, the space of return values is divided into four regions given by $\{A_{jk}; j = I(X_1 > u_1), k = I(X_2 > u_2)\}$ where $I(\cdot)$ is the indicator function. The method treats return observations below threshold as censored data. Finally, the dependence in extreme returns is modeled using a logistic function denoted by D_l.

The likelihood contribution corresponding to the observation of returns at time t (X_{1t}, X_{2t}) falling in region A_{jk} is denoted by $L_{jk}(X_{1t}, X_{2t})$ and given by:

$$L_{00}(X_{1t}, X_{2t}) = exp\big(-D_l(Y_1, Y_2)\big)$$

$$L_{01}(X_{1t}, X_{2t}) = \frac{\partial F_R^u(X_{1t}, X_{2t})}{\partial X_{2t}} = \exp\big(-D_l(Y_1, Z_2)\big)\frac{\partial D_l}{\partial X_{2t}}(Y_1, Z_2)K_2$$

$$L_{10}(X_{1t}, X_{2t}) = \frac{\partial F_R^u(X_{1t}, X_{2t})}{\partial X_{1t}} = exp\big(-D_l(Z_1, Y_2)\big)\frac{\partial D_l}{\partial X_{1t}}(Z_1, Y_2)K_1$$

$$L_{11}(X_{1t}, X_{2t}) = \frac{\partial^2 F_R^u(X_{1t}, X_{2t})}{\partial X_{1t}\partial X_{2t}}$$

$$= exp\big(-D_l(Z_1, Z_2)\big)\big(\frac{\partial D_l}{\partial X_{1t}}(Z_1, Z_2)\frac{\partial D_l}{\partial X_{2t}}(Z_1, Z_2) - \frac{\partial^2 D_1}{\partial X_{1t}\partial X_{2t}}(Z_1, Z_2)$$

where the variables Y_i Z_i and K_i for $i = 1,2$ are defined by:

$$Y_i = -1/\log F_{X_i}^{u_i}(u_i),$$

$$Z_i = -1/\log F_{X_i}^{u_i}(X_{it}),$$

$$K_i = -p_i\sigma_i^{-1}\,(1 + \xi_i(X_{it} - u_i)/\sigma_i)_+^{-(1+\xi_i)/\xi_i} Z_i^2 \exp(1/Z_i).$$

The likelihood contribution from the observation of returns at time t (X_{1t}, X_{2t}) for the bivariate distribution of return exceedances described by a set of parameters $\Phi = (p_1, p_2, \sigma_1, \sigma_2, \xi_1, \xi_2, \alpha)$ is given by:

$$L(X_{1t}, X_{2t}, \Phi) = \sum_{j,k\in\{0,1\}} L_{jk}(X_{1t}, X_{2t})I_{jk}(X_{1t}, X_{2t})$$

where $L_{jk}(X_{1t}, X_{2t})\ I\{(X_{1t}, X_{2t}) \in A_{jk}\}$. Hence, the likelihood for a set of T independent observations of returns is given by:

$$L(\{X_{1t}, X_{2t}\}_{t=1,T}, \Phi) = \prod_{t=1}^{T} L(X_{1t}, X_{2t}, \Phi).$$

Appendix 3. Computation of optimal threshold levels

Over a high threshold u the peaks-over-threshold approach constitutes an efficient method for modelling extremes via the GPD. However, one of the most important factors when dealing with extremes is the selection of the threshold u. If we select a low value of threshold, this will induce a significant estimation bias, by characterizing observations as exceedances not belonging to the distribution tails. On the other hand, if we select a high value of threshold u, this will lead to inefficiency by reducing the estimation sample and increasing standard errors. An optimal threshold optimizes the trade-off between inefficiency and sample bias.

In empirical literature, several approaches have been proposed to this issue. In this paper, we apply the procedure inspired by Gkillas et al. (2017) via the parametric bootstrap goodness-of-fit test of Villasenor-Alva and Gonzalez-Estrada (2009) for the computation of the optimal thresholds. Applying this procedure, we take into consideration the error for accepting that the GPD is a distribution for a random sample defined by a threshold u when u is not appropriate. We minimize this error via this powerful goodness-of-fit test. This test can provide results for the whole parameter space, in relation to other goodness-of-fit tests proposed in the literature (see Meintanis and Bassiakos, 2007; and Choulakian and Stephens, 2001). We describe this procedure in detail in this appendix.

Let $X = \{X_1, X_2, \ldots, X_n\}$ be a sequence of independent and identically distributed random variables defined on the positive real numbers with a continuous cumulative distribution function F_X, for $i = 1, 2, .., n$. Let also $X_{(1)} < X_{(2)} < \cdots < X_{(\mathrm{n})}$ be the corresponding order statistics. Our approach is proceeding in the following steps.

Step 1

We extract n subsequences from X, such that $X'_k = \left\{X'_{(k)}, X'_{(k+1)}, \ldots, X'_{(n)}\right\}^k \subseteq X$ for $k = 1, \ldots, n$, where k corresponds to a number of upper order statistic and can be associated with the unknown threshold u of the GPD.

Step 2

We apply an iterative *n*-step algorithm and we select the k that corresponds to the maximal *p*-value (p) of the intersection–union goodness-of-fit test of Villasenor-Alva and Gonzalez-Estrada (2009), as follows:

$$u = X'_{(k)} \widehat{=} \max_{k=1,\dots,n} \{p_{(k)}, p_{(k+1)} \dots, p_{(n)}\},\ k \in \{1, \dots, n\} \qquad \text{A3.1}$$

for the null hypothesis $H_0: F_X^u(x) \sim G_{\xi,\sigma}(x)$ defined by two sub-classes of GPD, the A^+ which corresponds that $H_0^+: F_X^u(x) \sim G_{\xi,\sigma}(x)$ with $\xi \geq 0$ and the A^- which corresponds that $H_0^-: F_X^u(x) \sim G_{\xi,\sigma}(x)$ with $\xi < 0$. Thus, $H_0: F \in (A^+ \cup A^-)$, which is rejected whenever both hypotheses H_0^+ and H_0^- are rejected.

Step 3

The optimal threshold u corresponds to the optimal k^{th} upper order statistic of the previous step.

Step 4

We apply this procedure in each distribution tail separately for 999 bootstrap samples.

Table 1. Estimation of the bivariate distribution of return exceedances for the European and US equity markets

Panel A: Negative return exceedances

				Parameters of the model						Wald tests	
p	u^{EU}	σ^{EU}	ξ^{EU}	u^{US}	σ^{US}	ξ^{US}	α	ρ	$H_0: \rho = 0$	$H_0: \rho = \rho_{nor}^{f.s}(u)$	$H_0: \rho = 1$
5%	0.036	0.015	0.057	0.031	0.026	-0.505	0.379	0.890	203.487	3.726	25.008
		(0.005)	(0.280)		(0.008)	(0.278)	(0.058)	(0.004)	[0.000]	[0.000]	[0.000]
10%	0.029	0.010	0.267	0.023	0.013	0.058	0.408	0.841	68.944	4.124	13.057
		(0.003)	(0.237)		(0.004)	(0.249)	(0.040)	(0.012)	[0.000]	[0.000]	[0.000]
20%	0.018	0.014	0.017	0.013	0.014	0.013	0.409	0.831	36.642	2.152	7.426
		(0.002)	(0.107)		(0.002)	(0.136)	(0.029)	(0.023)	[0.000]	[0.031]	[0.000]
30%	0.008	0.019	−0.099	0.007	0.015	−0.013	0.378	0.875	28.733	1.361	4.106
		(0.002)	(0.068)		(0.002)	(0.100)	(0.022)	(0.030)	[0.000]	[0.173]	[0.000]
40%	0.003	0.019	-0.087	0.003	0.014	0.009	0.365	0.877	26.008	0.839	3.638
		(0.002)	(0.064)		(0.002)	(0.088)	(0.018)	(0.034)	[0.000]	[0.400]	[0.000]
50%	0.000	0.019	-0.075	0.000	0.013	0.035	0.351	0.888	24.641	0.694	3.256
		(0.002)	(0.063)		(0.001)	(0.079)	(0.016)	(0.036)	[0.000]	[0.487]	[0.001]
3.01%	0.042	0.021	-0.130	0.041	0.030	0.026	0.349	0.878	63.419	3.130	8.678
3.00%		(0.008)	(0.288)		(0.009)	(0.000)	(0.071)	(0.014)	[0.000]	[0.001]	[0.000]

Panel B: Positive return exceedances

				Parameters of the model						Wald tests	
p	u^{EU}	σ^{EU}	ξ^{EU}	u^{US}	σ^{US}	ξ^{US}	α	ρ	$H_0: \rho = 0$	$H_0: \rho = \rho_{nor}^{f.s}(u)$	$H_0: \rho = 1$
50%	0.000	0.017	-0.151	0.000	0.014	-0.122	0.385	0.864	24.194	0.169	27.155
		(0.001)	(0.041)		(0.001)	(0.056)	(0.016)	(0.036)	[0.000]	[0.865]	[0.000]
40%	0.006	0.014	-0.094	0.004	0.012	-0.055	0.435	0.810	25.739	0.619	30.949
		(0.001)	(0.058)		(0.001)	(0.074)	(0.020)	(0.031)	[0.000]	[0.535]	[0.000]
30%	0.010	0.012	-0.050	0.008	0.011	-0.012	0.472	0.785	29.597	0.629	36.908
		(0.001)	(0.079)		(0.001)	(0.094)	(0.024)	(0.027)	[0.000]	[0.529]	[0.000]
20%	0.015	0.012	-0.067	0.012	0.011	-0.008	0.512	0.746	39.503	0.224	52.186
		(0.002)	(0.086)		(0.002)	(0.121)	(0.031)	(0.019)	[0.000]	[0.822]	[0.000]
10%	0.024	0.009	0.021	0.019	0.012	-0.072	0.566	0.686	153.940	0.486	223.625
		(0.002)	(0.132)		(0.003)	(0.165)	(0.043)	(0.004)	[0.000]	[0.626]	[0.000]
5%	0.031	0.008	0.094	0.027	0.012	-0.105	0.696	0.521	17.260	0.548	32.592
		(0.002)	(0.203)		(0.004)	(0.250)	(0.068)	(0.030)	[0.000]	[0.558]	[0.000]
2.00%	0.040	0.003	0.668	0.027	0.012	-0.105	0.795	0.384	6.003	1.379	15.239
5.01%		(0.002)	(0.538)		(0.004)	(0.250)	(0.087)	(0.064)	[0.000]	[0.168]	[0.000]

Note: this table gives the asymptotic maximum likelihood estimates of the parameters of the bivariate distribution of return exceedances for the European and US equity markets represented by the STOXX Europe 600 index and the S&P 500 index. Panel A reports the estimates for negative return exceedances. Panel B reports the estimates for positive return exceedances. Return exceedances are defined with a threshold u. Both fixed and optimal threshold levels are used for u. Fixed levels correspond to tail propabillity p: 5%, 10%, 20%, 30%, 40% and 50% (the same value of p is taken for both variables: $p = p^{EU} = p^{US}$). Optimal levels are computed by the procedure described in Appendix 1. They are given on the last line of each panel. Eight parameters are estimated: the threshold u associated with the tail probability p, the dispersion parameter σ, the tail index ξ for each series, the depedence parameter α of the logistic function used to model the tail dependence, and the correlation of return exceedances ρ (derived from the depedence parameter α). Standard errors are given below in parentheses. The null hypothesis of normality $H_0: \rho = \rho_{nor}$ is also tested by a Wald test. Two cases are considered: the asymptotic case and the finite-sample case. In the asymptotic case, the correlation of normal return exceedances over a threshold tending to infinity is theoretically equal to 0. In the finite-sample case, the correlation of return exceedances over a given finite threshold u, denoted by $\rho_{nor}^{f.s.}(u)$, is computed by simulation assuming that returns follow a bivariate normal distribution with parameters equal to the empirically-observed means and covariance matrix of returns. The issue of dependency is studied by considering two special cases: asymptotic independence $H_0: \rho = 0$ and total dependence $H_0: \rho = 1$. The p-values of the Wald tests are given below in brackets.

Table 2A. Estimation of the bivariate distribution of return exceedances for the European equity market and Bitcoin

Panel A: Negative return exceedances

Parameters of the model									Wald tests		
p	u^{EU}	σ^{EU}	ξ^{EU}	u^{BTC}	σ^{BTC}	ξ^{BTC}	α	ρ	$H_0: \rho = 0$	$H_0: \rho = \rho_{nor}^{f.s}(u)$	$H_0: \rho = 1$
5%	0.035	0.013	-0.084	0.193	0.073	-0.182	0.999	0.019	0.373	0.273	19.616
		(0.008)	(0.549)		(0.045)	(0.555)	(0.000)	(0.050)	[0.709]	[0.784]	[0.000]
10%	0.029	0.011	0.026	0.149	0.077	-0.176	0.923	0.170	20.099	1.988	97.826
		(0.003)	(0.246)		(0.022)	(0.219)	(0.040)	(0.008)	[0.000]	[0.047]	[0.000]
20%	0.016	0.019	-0.233	0.075	0.112	-0.288	0.868	0.234	23.400	0.426	76.600
		(0.003)	(0.121)		(0.020)	(0.124)	(0.034)	(0.010)	[0.000]	[0.795]	[0.000]
30%	0.007	0.022	-0.255	0.027	0.131	-0.312	0.801	0.364	33.791	0.343	59.062
		(0.003)	(0.096)		(0.018)	(0.092)	(0.030)	(0.011)	[0.000]	[0.732]	[0.000]
40%	0.002	0.021	-0.216	0.011	0.104	-0.166	0.735	0.472	25.625	0.417	28.636
		(0.003)	(0.087)		(0.015)	(0.100)	(0.026)	(0.018)	[0.000]	[0.677]	[0.000]
50%	0.000	0.021	-0.193	0.000	0.092	-0.086	0.715	0.477	21.525	2.316	23.640
		(0.003)	(0.086)		(0.013)	(0.106)	(0.023)	(0.022)	[0.000]	[0.021]	[0.000]
11.11%	0.028	0.009	0.136	0.160	0.063	-0.061	0.924	0.164	20.456	0.943	104.412
9.20%		(0.003)	(0.252)		(0.021)	(0.253)	(0.039)	(0.008)	[0.000]	[0.346]	[0.000]

Panel B: Positive return exceedances

Parameters of the model									Wald tests		
p	u^{EU}	σ^{EU}	ξ^{EU}	u^{BTC}	σ^{BTC}	ξ^{BTC}	α	ρ	$H_0: \rho = 0$	$H_0: \rho = \rho_{nor}^{f.s}(u)$	$H_0: \rho = 1$
50%	0.000	0.018	-0.295	0.000	0.094	-0.042	0.649	0.609	21.568	1.083	34.800
		(0.002)	(0.079)		(0.012)	(0.088)	(0.019)	(0.028)	[0.000]	[0.279]	[0.000]
40%	0.006	0.015	-0.242	0.024	0.099	-0.078	0.749	0.456	25.615	0.029	55.721
		(0.002)	(0.109)		(0.013)	(0.095)	(0.026)	(0.018)	[0.000]	[0.977]	[0.000]
30%	0.010	0.013	-0.200	0.050	0.099	-0.088	0.810	0.353	31.713	0.001	89.500
		(0.002)	(0.149)		(0.016)	(0.111)	(0.030)	(0.011)	[0.000]	[1.000]	[0.000]
20%	0.014	0.016	-0.397	0.089	0.095	-0.084	0.868	0.265	105.568	0.555	398.491
		(0.002)	(0.126)		(0.019)	(0.142)	(0.034)	(0.003)	[0.000]	[0.579]	[0.000]
10%	0.025	0.012	-0.437	0.155	0.075	0.051	0.898	0.209	15.893	2.771	75.683
		(0.000)	(0.077)		(0.025)	(0.274)	(0.044)	(0.013)	[0.000]	[0.006]	[0.000]
5%	0.031	0.013	-0.716	0.202	0.139	-0.470	0.949	0.084	3.896	0.686	46.574
		(0.000)	(0.060)		(0.081)	(0.530)	(0.047)	(0.021)	[0.000]	[0.492]	[0.000]
11.11%	0.023	0.161	0.080	0.114	0.111	0.013	-0.465	0.175	65.110	0.113	371.015
11.14%		(0.023)	(0.019)		(0.019)	(0.000)	(0.065)	(0.003)	[0.000]	[0.910]	[0.000]

Note: this table gives the asymptotic maximum likelihood estimates of the parameters of the bivariate distribution of return exceedances for the European equity market, represented by the STOXX Europe 600 index, and Bitcoin. Panel A reports the estimates for negative return exceedances. Panel B reports the estimates for positive return exceedances. Return exceedances are defined with a threshold u. Both fixed and optimal threshold levels are used for u. Fixed levels correspond to tail propabillity p: 5%, 10%, 20%, 30%, 40% and 50% (the same value of p is taken for both variables: $p = p^{EU} = p^{BTC}$). Optimal levels are computed by the procedure described in Appendix 1. They are given on the last line of each panel. Eight parameters are estimated: the threshold u associated with the tail probability p, the dispersion parameter σ, the tail index ξ for each series, the depedence parameter α of the logistic function used to model the tail dependence, and the correlation of return exceedances ρ (derived from the depedence parameter α). Standard errors are given below in parentheses. The null hypothesis of normality $H_0: \rho = \rho_{nor}$ is also tested by a Wald test. Two cases are considered: the asymptotic case and the finite-sample case. In the asymptotic case, the correlation of normal return exceedances over a threshold tending to infinity is theoretically equal to 0. In the finite-sample case, the correlation of return exceedances over a given finite threshold u, denoted by $\rho_{nor}^{f.s.}(u)$, is computed by simulation assuming that returns follow a bivariate normal distribution with parameters equal to the empirically-observed means and covariance matrix of returns. The issue of dependency is studied by considering two special cases: asymptotic independence $H_0: \rho = 0$ and total dependence $H_0: \rho = 1$. The p-values of the Wald tests are given below in brackets.

Table 2B. Estimation of the bivariate distribution of return exceedances for the US equity market and Bitcoin

Panel A: Negative return exceedances

Parameters of the model									Wald tests		
p	u^{US}	σ^{US}	ξ^{US}	u^{BTC}	σ^{BTC}	ξ^{BTC}	α	ρ	$H_0: \rho = 0$	$H_0: \rho = \rho_{nor}^{f.s}(u)$	$H_0: \rho = 1$
5%	0.031	0.026	-0.505	0.193	0.073	-0.182	0.936	0.123	10.250	2.987	73.083
		(0.008)	(0.278)		(0.044)	(0.554)	(0.038)	(0.012)	[0.000]	[0.002]	[0.000]
10%	0.019	0.019	-0.237	0.149	0.077	-0.176	0.919	0.186	17.183	0.368	75.152
		(0.006)	(0.259)		(0.022)	(0.219)	(0.041)	(0.011)	[0.000]	[0.713]	[0.000]
20%	0.012	0.010	0.172	0.075	0.112	-0.288	0.827	0.333	444.804	2.343	890.161
		(0.002)	(0.207)		(0.020)	(0.124)	(0.036)	(0.001)	[0.000]	[0.019]	[0.000]
30%	0.006	0.013	0.011	0.027	0.131	-0.312	0.771	0.414	37.986	1.443	53.672
		(0.002)	(0.125)		(0.018)	(0.092)	(0.031)	(0.011)	[0.000]	[0.149]	[0.000]
40%	0.002	0.013	0.030	0.011	0.104	-0.166	0.720	0.499	27.258	0.376	27.311
		(0.002)	(0.110)		(0.015)	(0.100)	(0.026)	(0.018)	[0.000]	[0.707]	[0.000]
50%	0.000	0.012	0.036	0.000	0.092	-0.086	0.686	0.547	23.502	0.761	19.614
		(0.002)	(0.099)		(0.01)3	(0.106)	(0.023)	(0.023)	[0.000]	[0.446]	[0.000]
7.28%	0.024	0.018	-0.248	0.160	0.063	-0.061	0.904	0.192	9.803	1.215	41.341
9.20%		(0.008)	(0.397)		(0.021)	(0.253)	(0.048)	(0.020)	[0.020]	[0.225]	[0.000]

Panel B: Positive return exceedances

Parameters of the model									Wald tests		
p	u^{US}	σ^{US}	ξ^{US}	u^{BTC}	σ^{BTC}	ξ^{BTC}	α	ρ	$H_0: \rho = 0$	$H_0: \rho = \rho_{nor}^{f.s}(u)$	$H_0: \rho = 1$
50%	0.000	0.015	-0.322	0.000	0.094	-0.042	0.657	0.599	22.639	0.442	37.187
		(0.002)	(0.075)		(0.012)	(0.088)	(0.021)	(0.026)	[0.000]	[0.658]	[0.000]
40%	0.005	0.012	-0.252	0.024	0.099	-0.078	0.710	0.514	27.222	0.751	52.489
		(0.002)	(0.102)		(0.013)	(0.095)	(0.026)	(0.019)	[0.000]	[0.453]	[0.000]
30%	0.008	0.012	-0.285	0.050	0.099	-0.088	0.835	0.312	30.213	2.012	96.607
		(0.002)	(0.120)		(0.016)	(0.111)	(0.029)	(0.010)	[0.000]	[0.044]	[0.000]
20%	0.012	0.012	-0.326	0.089	0.095	-0.084	0.851	0.293	249.830	0.389	853.126
		(0.002)	(0.156)		(0.019)	(0.142)	(0.035)	(0.001)	[0.000]	[0.697]	[0.000]
10%	0.018	0.012	-0.514	0.155	0.075	0.051	0.869	0.260	14.893	2.940	56.938
		(0.000)	(0.063)		(0.025)	(0.274)	(0.048)	(0.017)	[0.000]	[0.003]	[0.000]
5%	0.025	0.011	-0.665	0.202	0.139	-0.470	0.901	0.200	5.321	0.731	26.387
		(0.000)	(0.074)		(0.081)	(0.530)	(0.060)	(0.038)	[0.000]	[0.465]	[0.000]
6.00%	0.024	0.008	-0.356	0.182	0.125	-0.331	0.916	0.167	6.139	0.074	36.674
6.10%		(0.000)	(0.127)		(0.057)	(0.394)	(0.052)	(0.027)	[0.000]	[0.941]	[0.000]

Note: this table gives the asymptotic maximum likelihood estimates of the parameters of the bivariate distribution of return exceedances for the US equity markets represented by the S&P 500 index and Bitcoin. Panel A reports the estimates for negative return exceedances. Panel B reports the estimates for positive return exceedances. Return exceedances are defined with a threshold u. Both fixed and optimal threshold levels are used for u. Fixed levels correspond to tail propabillity p: 5%, 10%, 20%, 30%, 40% and 50% (the same value of p is taken for both variables: $p = p^{US} = p^{BTC}$). Optimal levels are computed by the procedure described in Appendix 1. They are given on the last line of each panel. Eight parameters are estimated: the threshold u associated with the tail probability p, the dispersion parameter σ, the tail index ξ for each series, the depedence parameter α of the logistic function used to model the tail dependence, and the correlation of return exceedances ρ (derived from the depedence parameter α). Standard errors are given below in parentheses. The null hypothesis of normality $H_0: \rho = \rho_{nor}$ is also tested by a Wald test. Two cases are considered: the asymptotic case and the finite-sample case. In the asymptotic case, the correlation of normal return exceedances over a threshold tending to infinity is theoretically equal to 0. In the finite-sample case, the correlation of return exceedances over a given finite threshold u, denoted by $\rho_{nor}^{f.s.}(u)$, is computed by simulation assuming that returns follow a bivariate normal distribution with parameters equal to the empirically-observed means and covariance matrix of returns. The issue of dependency is studied by considering two special cases: asymptotic independence $H_0: \rho = 0$ and total dependence $H_0: \rho = 1$. The p-values of the Wald tests are given below in brackets.

Table 3A. Estimation of the bivariate distribution of return exceedances for the European equity market and Gold

Panel A: Negative return exceedances

	Parameters of the model								Wald tests		
p	u^{EU}	σ^{EU}	ξ^{EU}	u^{Gold}	σ^{Gold}	ξ^{Gold}	α	ρ	$H_0: \rho = 0$	$H_0: \rho = \rho_{nor}^{f.s}(u)$	$H_0: \rho = 1$
5%	0.036	0.015	0.057	0.033	0.023	-0.260	0.965	0.060	62.609	61.615	977.029
		(0.005)	(0.280)		(0.007)	(0.244)	(0.033)	(0.001)	[0.000]	[0.000]	[0.000]
10%	0.029	0.010	0.267	0.025	0.015	-0.017	0.915	0.167	176.997	1.592	882.646
		(0.003)	(0.237)		(0.004)	(0.184)	(0.033)	(0.001)	[0.000]	[0.111]	[0.000]
20%	0.018	0.014	0.017	0.019	0.010	0.178	0.901	0.201	15.461	0.734	61.461
		(0.002)	(0.107)		(0.002)	(0.150)	(0.025)	(0.013)	[0.000]	[0.462]	[0.000]
30%	0.008	0.019	-0.099	0.011	0.016	-0.058	0.817	0.353	19.397	0.139	35.490
		(0.002)	(0.068)		(0.002)	(0.081)	(0.024)	(0.018)	[0.000]	[0.890]	[0.000]
40%	0.003	0.019	-0.087	0.004	0.019	-0.124	0.750	0.460	19.210	0.574	22.557
		(0.002)	(0.064)		(0.002)	(0.062)	(0.021)	(0.024)	[0.000]	[0.566]	[0.000]
50%	0.000	0.019	-0.075	0.000	0.021	-0.152	0.707	0.522	18.782	0.779	17.599
		(0.002)	(0.063)		(0.002)	(0.054)	(0.018)	(0.028)	[0.000]	[0.436]	[0.000]
3.01%	0.042	0.030	0.021	0.025	0.111	0.014	0.936	0.123	0.135	0.033	11.580
10.04%		(0.009)	(0.008)		(0.016)	(0.003)	(0.041)	(0.077)	[0.012]	[0.016]	[0.000]

Panel B: Positive return exceedances

	Parameters of the model								Wald tests		
p	u^{EU}	σ^{EU}	ξ^{EU}	u^{Gold}	σ^{Gold}	ξ^{Gold}	α	ρ	$H_0: \rho = 0$	$H_0: \rho = \rho_{nor}^{f.s}(u)$	$H_0: \rho = 1$
50%	0.000	0.017	-0.151	0.000	0.021	-0.285	0.638	0.606	19.628	1.461	31.804
		(0.001)	(0.041)		(0.001)	(0.034)	(0.017)	(0.031)	(0.000)	(0.144)	(0.000)
40%	0.006	0.014	-0.094	0.006	0.016	-0.173	0.726	0.473	20.201	0.245	42.256
		(0.001)	(0.058)		(0.002)	(0.062)	(0.022)	(0.023)	(0.000)	(0.807)	(0.000)
30%	0.010	0.012	-0.050	0.012	0.012	-0.055	0.773	0.411	23.845	1.541	57.633
		(0.001)	(0.079)		(0.002)	(0.094)	(0.026)	(0.017)	(0.000)	(0.123)	(0.000)
20%	0.015	0.012	-0.067	0.024	0.012	-0.064	0.817	0.341	42.200	4.619	123.536
		(0.002)	(0.086)		(0.003)	(0.161)	(0.031)	(0.008)	(0.000)	(0.000)	(0.000)
10%	0.024	0.009	0.021	0.025	0.013	-0.118	0.801	0.366	44.033	7.837	120.099
		(0.002)	(0.132)		(0.003)	(0.159)	(0.044)	(0.008)	(0.000)	(0.000)	(0.000)
5%	0.031	0.008	0.094	0.033	0.009	0.111	0.796	0.372	12.308	12.280	32.711
		(0.002)	(0.203)		(0.003)	(0.344)	(0.061)	(0.030)	(0.000)	(0.000)	(0.000)
2.02%	0.040	0.003	0.668	0.028	0.011	-0.023	0.855	0.270	0.286	0.001	6.507
8.08%		(0.002)	(0.538)		(0.003)	(0.204)	(0.068)	(0.118)	[0.044]	[0.000]	[0.000]

Note: this table gives the asymptotic maximum likelihood estimates of the parameters of the bivariate distribution of return exceedances for the European equity market, represented by the STOXX Europe 600 index, and Gold. Panel A reports the estimates for negative return exceedances. Panel B reports the estimates for positive return exceedances. Return exceedances are defined with a threshold u. Both fixed and optimal threshold levels are used for u. Fixed levels correspond to tail propabillity p: 5%, 10%, 20%, 30%, 40% and 50% (the same value of p is taken for both variables: $p = p^{EU} = p^{Gold}$). Optimal levels are computed by the procedure described in Appendix 1. They are given on the last line of each panel. Eight parameters are estimated: the threshold u associated with the tail probability p, the dispersion parameter σ, the tail index ξ for each series, the depedence parameter α of the logistic function used to model the tail dependence, and the correlation of return exceedances ρ (derived from the depedence parameter α). Standard errors are given below in parentheses. The null hypothesis of normality $H_0: \rho = \rho_{nor}$ is also tested by a Wald test. Two cases are considered: the asymptotic case and the finite-sample case. In the asymptotic case, the correlation of normal return exceedances over a threshold tending to infinity is theoretically equal to 0. In the finite-sample case, the correlation of return exceedances over a given finite threshold u, denoted by $\rho_{nor}^{f.s.}(u)$, is computed by simulation assuming that returns follow a bivariate normal distribution with parameters equal to the empirically-observed means and covariance matrix of returns. The issue of dependency is studied by considering two special cases: asymptotic independence $H_0: \rho = 0$ and total dependence $H_0: \rho = 1$. The p-values of the Wald tests are given below in brackets.

Table 3B. Estimation of the bivariate distribution of return exceedances for the US equity market and Gold

Panel A: Negative return exceedances

			Parameters of the model						Wald tests		
p	u^{US}	σ^{US}	ξ^{US}	u^{Gold}	σ^{Gold}	ξ^{Gold}	α	ρ	$H_0: \rho = 0$	$H_0: \rho = \rho_{nor}^{f.s}(u)$	$H_0: \rho = 1$
5%	0.031	0.026	-0.505	0.033	0.023	-0.260	0.966	0.089	9.888	2.842	101.222]
		(0.008)	(0.278)		(0.007)	(0.244)	(0.032)	(0.009)	[0.000]	[0.004]	[0.000]
10%	0.023	0.013	0.058	0.025	0.015	-0.017	0.901	0.193	193.000	16.696	806.998
		(0.004)	(0.249)		(0.004)	(0.184)	(0.035)	(0.001)	[0.000]	[0.000]	[0.000]
20%	0.013	0.014	0.013	0.019	0.010	0.178	0.877	0.237	19.890	0.000	64.008
		(0.002)	(0.136)		(0.002)	(0.150)	(0.027)	(0.012)	[0.000]	[1.000]	[0.000]
30%	0.007	0.015	-0.013	0.011	0.016	-0.058	0.779	0.415	22.105	2.085	31.210
		(0.002)	(0.100)		(0.002)	(0.081)	(0.024)	(0.019)	[0.000]	[0.037]	[0.000]
40%	0.002	0.013	0.045	0.000	0.021	-0.162	0.708	0.515	19.214	0.214	18.119
		(0.002)	(0.087)		(0.002)	(0.053)	(0.019)	(0.027)	[0.000]	[0.830]	[0.000]
50%	0.000	0.013	0.035	0.000	0.021	-0.152	0.671	0.558	19.045	0.803	15.056
		(0.001)	(0.079)		(0.002)	(0.054)	(0.018)	(0.029)	[0.000]	[0.422]	[0.000]
6.06%	0.028	0.024	-0.394	0.025	0.015	-0.017	0.934	0.189	188.750	16.401	810.9120
10.01%		(0.008)	(0.261)		(0.004)	(0.184)	(0.035)	(0.001)	[0.000]	[0.000]	[0.000]

Panel B: Positive return exceedances

			Parameters of the model						Wald tests		
p	u^{US}	σ^{US}	ξ^{US}	u^{Gold}	σ^{Gold}	ξ^{Gold}	α	ρ	$H_0: \rho = 0$	$H_0: \rho = \rho_{nor}^{f.s}(u)$	$H_0: \rho = 1$
50%	0.000	0.014	-0.122	0.000	0.021	-0.285	0.631	0.614	20.258	2.070	32.393
		(0.001)	(0.056)		(0.001)	(0.034)	(0.018)	(0.030)	[0.000]	[0.038]	[0.000]
40%	0.004	0.012	-0.055	0.006	0.016	-0.173	0.695	0.516	21.108	1.730	40.370
		(0.001)	(0.074)		(0.002)	(0.062)	(0.022)	(0.024)	[0.000]	[0.084]	[0.000]
30%	0.008	0.011	-0.012	0.012	0.012	-0.055	0.745	0.453	24.473	3.195	53.530
		(0.001)	(0.094)		(0.002)	(0.094)	(0.025)	(0.019)	[0.000]	[0.001]	[0.000]
20%	0.012	0.011	-0.008	0.016	0.014	-0.144	0.819	0.338	36.281	3.579	107.008
		(0.002)	(0.121)		(0.002)	(0.099)	(0.030)	(0.009)	[0.000]	[0.000]	[0.000]
10%	0.019	0.012	-0.072	0.025	0.013	-0.118	0.856	0.274	42.145	8.827	153.822
		(0.003)	(0.165)		(0.003)	(0.159)	(0.041)	(0.006)	[0.000]	[0.000]	[0.000
5%	0.027	0.012	-0.105	0.033	0.009	0.111	0.864	0.259	9.477	5.027	36.304
		(0.004)	(0.250)		(0.003)	(0.344)	(0.055)	(0.027)	[0.000]	[0.000]	[0.000]
2.70%	0.027	0.012	-0.105	0.029	0.012	-0.083	0.855	0.269	0.286	0.030	12.789
7.07%		(0.004)	(0.250)		(0.003)	(0.213)	(0.052)	(0.090)	[0.022]	[0.015]	[0.000]

Note: this table gives the asymptotic maximum likelihood estimates of the parameters of the bivariate distribution of return exceedances for the US equity markets represented by the S&P 500 index and Gold. Panel A reports the estimates for negative return exceedances. Panel B reports the estimates for positive return exceedances. Return exceedances are defined with a threshold u. Both fixed and optimal threshold levels are used for u. Fixed levels correspond to tail propabillity p: 5%, 10%, 20%, 30%, 40% and 50% (the same value of p is taken for both variables: $p = p^{US} = p^{Gold}$). Optimal levels are computed by the procedure described in Appendix 1. They are given on the last line of each panel. Eight parameters are estimated: the threshold u associated with the tail probability p, the dispersion parameter σ, the tail index ξ for each series, the depedence parameter α of the logistic function used to model the tail dependence, and the correlation of return exceedances ρ (derived from the depedence parameter α). Standard errors are given below in parentheses. The null hypothesis of normality $H_0: \rho = \rho_{nor}$ is also tested by a Wald test. Two cases are considered: the asymptotic case and the finite-sample case. In the asymptotic case, the correlation of normal return exceedances over a threshold tending to infinity is theoretically equal to 0. In the finite-sample case, the correlation of return exceedances over a given finite threshold u, denoted by $\rho_{nor}^{f.s.}(u)$, is computed by simulation assuming that returns follow a bivariate normal distribution with parameters equal to the empirically-observed means and covariance matrix of returns. The issue of dependency is studied by considering two special cases: asymptotic independence $H_0: \rho = 0$ and total dependence $H_0: \rho = 1$. The p-values of the Wald tests are given below in brackets.

Table 4. Estimation of the bivariate distribution of return exceedances for Bitcoin and Gold

Panel A: Negative return exceedances

	Parameters of the model								Wald tests		
p	u^{BTC}	σ^{BTC}	ξ^{BTC}	u^{Gold}	σ^{Gold}	ξ^{Gold}	α	ρ	$H_0: \rho = 0$	$H_0: \rho = \rho_{nor}^{f.s}(u)$	$H_0: \rho = 1$
5%	0.193	0.073	-0.182	0.032	0.024	-0.563	0.949	0.083	3.934	0.698	43.317
		(0.045)	(0.555)		(0.013)	(0.561)	(0.047)	(0.021)	[0.000]	[0.485]	[0.000]
10%	0.149	0.077	-0.176	0.023	0.016	-0.155	0.973	0.049	5.061	2.663	99.065
		(0.022)	(0.219)		(0.005)	(0.250)	(0.026)	(0.010)	[0.000]	[0.008]	[0.000]
20%	0.075	0.112	-0.288	0.017	0.011	0.076	0.860	0.254	82.112	0.492	241.551
		(0.020)	(0.124)		(0.002)	(0.168)	(0.034)	(0.003)	[0.000]	[0.623]	[0.000]
30%	0.027	0.131	-0.312	0.010	0.015	-0.094	0.790	0.394	34.593	1.075	53.109
		(0.018)	(0.092)		(0.002)	(0.105)	(0.030)	(0.011)	[0.000]	[0.282]	[0.000]
40%	0.011	0.104	-0.166	0.004	0.018	-0.170	0.740	0.462	25.310	0.372	29.532
		(0.015)	(0.100)		(0.002)	(0.080)	(0.026)	(0.018)	[0.000]	[0.710]	[0.000]
50%	0.000	0.092	-0.086	0.000	0.020	-0.208	0.698	0.520	22.429	1.240	20.688
		(0.013)	(0.106)		(0.002)	(0.069)	(0.023)	(0.023)	[0.000]	[0.215]	[0.000]
11.03%	0.137	0.087	-0.231	0.022	0.014	-0.065	0.952	0.054	22.662	2.962	399.423
11.00%		(0.023)	(0.195)		(0.004)	(0.246)	(0.031)	(0.002)	[0.000]	[0.003]	[0.000]

Panel B: Positive return exceedances

	Parameters of the model								Wald tests		
p	u^{BTC}	σ^{BTC}	ξ^{BTC}	u^{Gold}	σ^{Gold}	ξ^{Gold}	α	ρ	$H_0: \rho = 0$	$H_0: \rho = \rho_{nor}^{f.s}(u)$	$H_0: \rho = 1$
50%	0.000	0.094	-0.042	0.000	0.018	-0.215	0.648	0.590	21.983	0.553	36.643
		(0.012)	(0.088)		(0.002)	(0.051)	(0.020)	(0.027)	[0.000]	[0.580]	[0.000
40%	0.024	0.099	-0.078	0.006	0.016	-0.192	0.722	0.492	27.353	1.224	55.076
		(0.013)	(0.095)		(0.002)	(0.062)	(0.026)	(0.018)	[0.000]	[0.221]	[0.000]
30%	0.050	0.099	-0.088	0.012	0.012	-0.104	0.781	0.385	35.816	0.792	92.725
		(0.016)	(0.111)		(0.002)	(0.089)	(0.031)	(0.011)	[0.000]	[0.428]	[0.000]
20%	0.089	0.095	-0.084	0.016	0.011	-0.089	0.829	0.331	296.471	2.485	896.122
		(0.019)	(0.142)		(0.002)	(0.112)	(0.036)	(0.001)	[0.000]	[0.013]	[0.000]
10%	0.155	0.075	0.051	0.024	0.009	0.030	0.919	0.164	15.118	0.393	91.890
		(0.025)	(0.274)		(0.002)	(0.205)	(0.041)	(0.011)	[0.000]	[0.694]	[0.000]
5%	0.202	0.139	-0.470	0.032	0.006	0.249	0.948	0.106	4.740	1.167	44.464
		(0.081)	(0.530)		(0.002)	(0.363)	(0.048)	(0.022)	[0.000]	[0.243]	[0.000]
10.34%	0.155	0.067	0.125	0.038	0.004	0.586	0.999	0.024	0.478	1.093	19.936
2.29%		(0.023)	(0.285)		(0.002)	(0.649)	(0.000)	(0.050)	[0.633]	[0.274]	[0.000]

Note: this table gives the asymptotic maximum likelihood estimates of the parameters of the bivariate distribution of return exceedances for Bitcoin and Gold. Panel A reports the estimates for negative return exceedances. Panel B reports the estimates for positive return exceedances. Return exceedances are defined with a threshold u. Both fixed and optimal threshold levels are used for u. Fixed levels correspond to tail propabillity p: 5%, 10%, 20%, 30%, 40% and 50% (the same value of p is taken for both variables: $p = p^{BTC} = p^{Gold}$). Optimal levels are computed by the procedure described in Appendix 1. They are given on the last line of each panel. Eight parameters are estimated: the threshold u associated with the tail probability p, the dispersion parameter σ, the tail index ξ for each series, the depedence parameter α of the logistic function used to model the tail dependence, and the correlation of return exceedances ρ (derived from the depedence parameter α). Standard errors are given below in parentheses. The null hypothesis of normality $H_0: \rho = \rho_{nor}$ is also tested by a Wald test. Two cases are considered: the asymptotic case and the finite-sample case. In the asymptotic case, the correlation of normal return exceedances over a threshold tending to infinity is theoretically equal to 0. In the finite-sample case, the correlation of return exceedances over a given finite threshold u, denoted by $\rho_{nor}^{f.s.}(u)$, is computed by simulation assuming that returns follow a bivariate normal distribution with parameters equal to the empirically-observed means and covariance matrix of returns. The issue of dependency is studied by considering two special cases: asymptotic independence $H_0: \rho = 0$ and total dependence $H_0: \rho = 1$. The p-values of the Wald tests are given below in brackets.

Table 5. Comparative results for equity markets, Bitcoin and Gold

Panel A: Correlation among return exceedances for the European equity market, Bitcoin and Gold

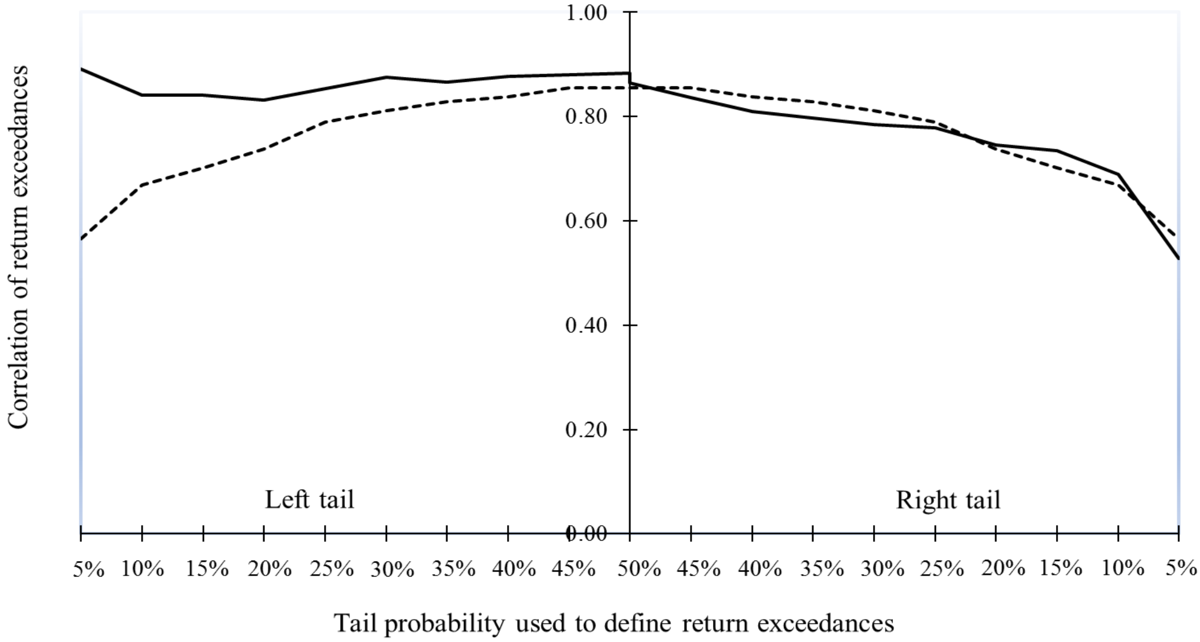
Negative return exceedances				Positive return exceedances			
Parameters			Wald test	Parameters			Wald test
p	$\rho^{EU/BTC}$	$\rho^{EU/Gold}$	$H_0: \rho^{EU/BTC} = \rho^{EU/Gold}$	p	$\rho^{EU/BTC}$	$\rho^{EU/Gold}$	$H_0: \rho^{EU/BTC} = \rho^{EU/Gold}$
5%	0.019	0.060	0.804	5%	0.084	0.372	5.647
	(0.050)	(0.001)	[0.421]		(0.021)	(0.030)	[0.000]
10%	0.170	0.167	0.333	10%	0.209	0.366	7.476
	(0.008)	(0.001)	[0.739]		(0.013)	(0.008)	[0.000]
20%	0.234	0.201	1.434	20%	0.265	0.341	6.909
	(0.010)	(0.013)	[0.1513]		(0.003)	(0.008)	[0.000]
30%	0.364	0.353	0.379	30%	0.353	0.411	2.071
	(0.011)	(0.018)	[0.704]		(0.011)	(0.017)	[0.038]
40%	0.472	0.460	0.286	40%	0.456	0.473	0.415
	(0.018)	(0.024)	[0.775]		(0.018)	(0.023)	[0.678]
50%	0.477	0.522	0.900	50%	0.609	0.606	0.051
	(0.022)	(0.028)	[0.368]		(0.028)	(0.031)	[0.959]
Optimal thresholds	0.164	0.123	0.482	Optimal thresholds	0.175	0.270	0.785
	(0.008)	(0.077)	[0.630]		(0.003)	(0.118)	[0.432]

Panel B: Correlation among return exceedances for the US equity market, Bitcoin and Gold

Negative return exceedances				Positive return exceedances			
Parameters			Wald test	Parameters			Wald test
p	$\rho^{US/BTC}$	$\rho^{US/Gold}$	$H_0: \rho^{US/BTC} = \rho^{US/Gold}$	p	$\rho^{US/BTC}$	$\rho^{US/Gold}$	$H_0: \rho^{US/BTC} = \rho^{US/Gold}$
5%	0.123	0.089	1.619	5%	0.200	0.259	0.908
	(0.012)	(0.009)	[0.105]		(0.038)	(0.027)	[0.364]
10%	0.186	0.193	0.738	10%	0.260	0.274	0.609
	(0.011)	(0.001)	[0.333]		(0.017)	(0.006)	[0.543]
20%	0.333	0.237	7.385	20%	0.293	0.338	4.500
	(0.001)	(0.012)	[0.000]		(0.001)	(0.009)	[0.000]
30%	0.414	0.415	0.033	30%	0.312	0.453	4.862
	(0.011)	(0.019)	[0.973]		(0.010)	(0.019)	[0.000]
40%	0.499	0.469	0.714	40%	0.514	0.516	0.047
	(0.018)	(0.024)	[0.475]		(0.019)	(0.024)	[0.963]
50%	0.547	0.558	0.212	50%	0.599	0.614	0.268
	(0.023)	(0.029)	[0.832]		(0.026)	(0.030)	[0.789]
Optimal thresholds	0.192	0.189	0.886	Optimal thresholds	0.167	0.269	0.872
	(0.020)	(0.001)	[0.142]		(0.027)	(0.090)	[0.383]

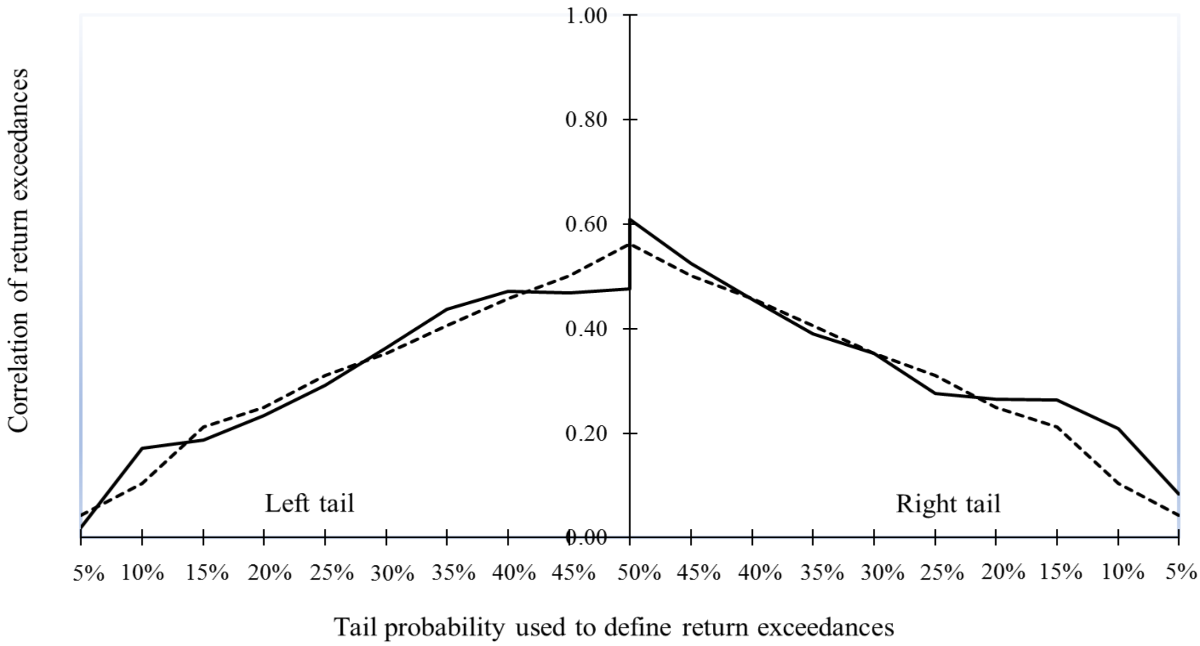
Note: this table compares the results for equity markets including Bitcoin or Gold. Panel A reports the correlation between return exceedances for the European equity market and Bitcoin, and the European equity market and Gold. Panel B reports the correlation between return exceedances for the US equity market and Bitcoin, and the US equity market and Gold. For a given estimation, the same value of tail probability p is taken for the four variables: $p = p^{EU} = p^{US} = p^{BTC} = p^{Gold}$. Standard errors are given below in parentheses. The null hypotheses of equal correlation of return exceedances $H_0: \rho^{EU/BTC} = \rho^{EU/Gold}$ and $H_0: \rho^{US/BTC} = \rho^{US/Gold}$ are also tested by a Wald test. The p-values of the test are given below in brackets.

Figure 1. Correlation between return exceedances for the European and US equity markets



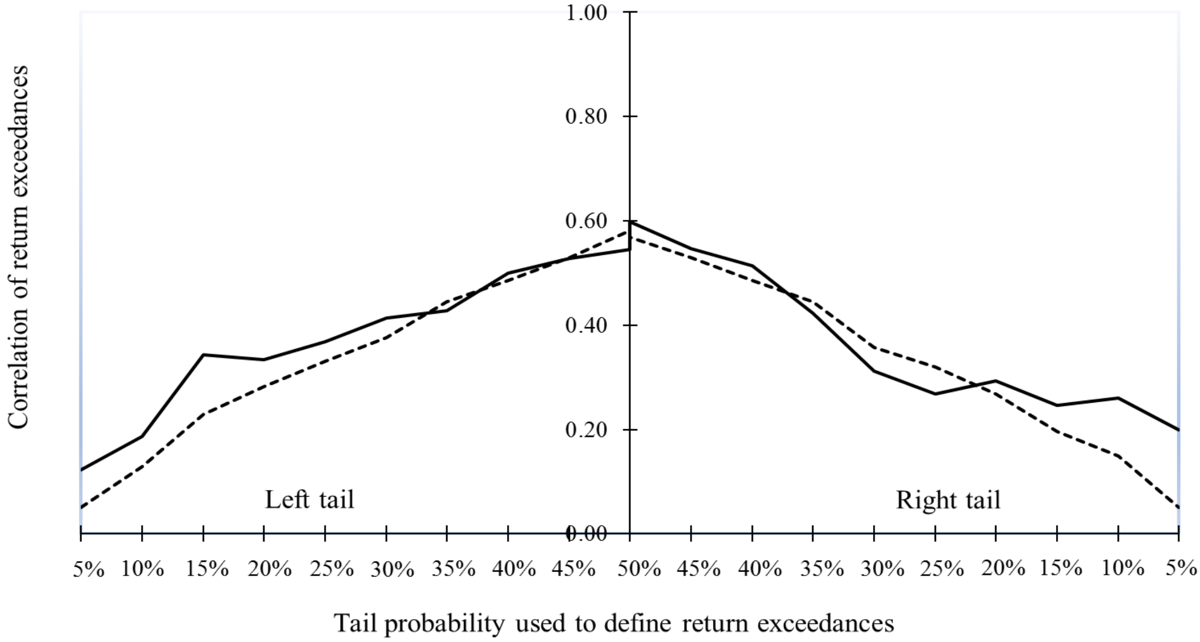
Note: this figure represents the correlation of return exceedances between the European and US equity markets represented by the STOXX Europe 600 index and the S&P 500 index. The *solid line* represents the correlation between actual return exceedances obtained from the estimation of the bivariate distribution modeled with the logistic function (see the estimation results in Table 1). The *dotted line* represents the theoretical correlation between simulated normal return exceedances assuming a bivariate normal distribution with parameters equal to the empirically-observed means and covariance matrix of returns. The value of the tail probability p used to define the threshold for return exceedances ranges from 5% to 50% for both negative return exceedances (left tail) and positive return exceedances (right tail). For a given estimation, the same value of p is taken for both variables: $p = p^{EU} = p^{US}$.

Figure 2A. Correlation between return exceedances for the European equity market and Bitcoin



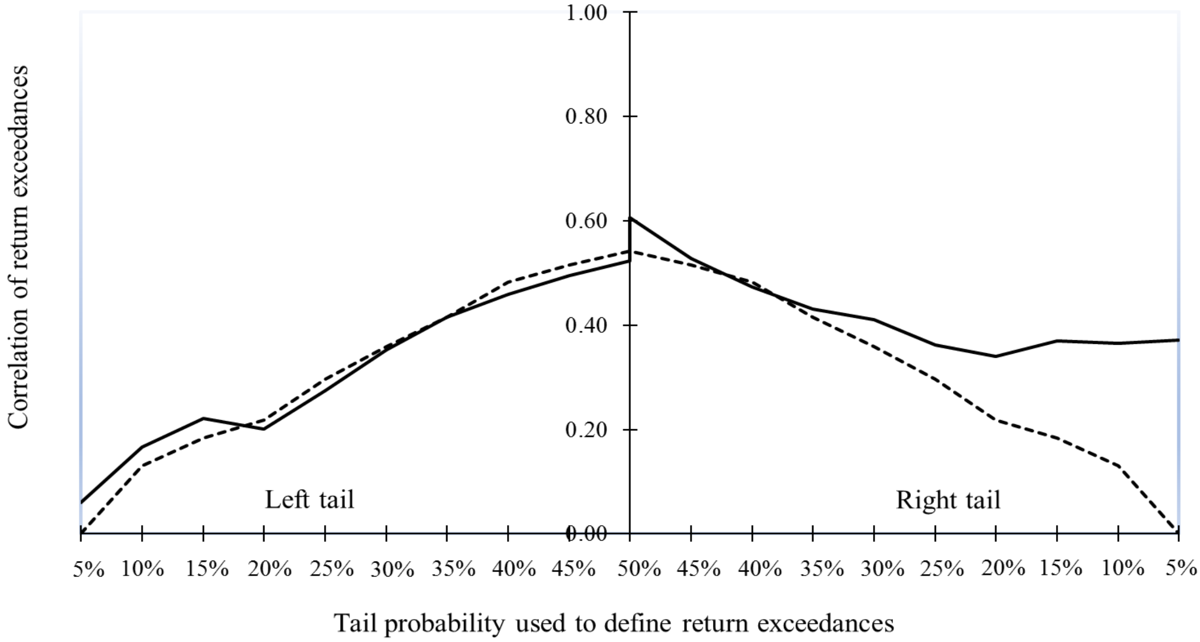
Note: this figure represents the correlation of return exceedances between the European equity markets represented by the STOXX Europe 600 index and Bitcoin. The *solid line* represents the correlation between actual return exceedances obtained from the estimation of the bivariate distribution modeled with the logistic function (see the estimation results in Table 2A). The *dotted line* represents the theoretical correlation between simulated normal return exceedances assuming a bivariate normal distribution with parameters equal to the empirically-observed means and covariance matrix of returns. The value of the tail probability p used to define the threshold for return exceedances ranges from 5% to 50% for both negative return exceedances (left tail) and positive return exceedances (right tail). For a given estimation, the same value of p is taken for both variables: $p = p^{EU} = p^{BTC}$.

Figure 2B. Correlation between return exceedances for the US equity market and Bitcoin



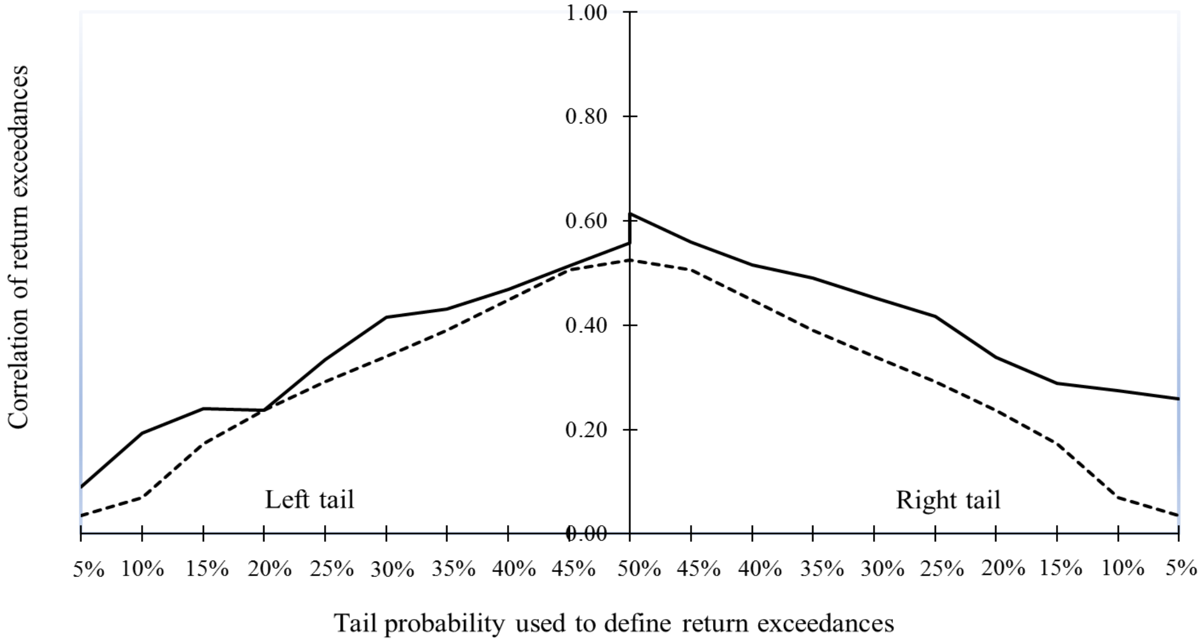
Note: this figure represents the correlation of return exceedances between the US equity markets represented by the S&P 500 index and Bitcoin. The *solid line* represents the correlation between actual return exceedances obtained from the estimation of the bivariate distribution modeled with the logistic function (see the estimation results in Table 2B). The *dotted line* represents the theoretical correlation between simulated normal return exceedances assuming a bivariate normal distribution with parameters equal to the empirically-observed means and covariance matrix of returns. The value of the tail probability p used to define the threshold for return exceedances ranges from 5% to 50% for both negative return exceedances (left tail) and positive return exceedances (right tail). For a given estimation, the same value of p is taken for both variables: $p = p^{US} = p^{BTC}$.

Figure 3A. Correlation between return exceedances for the European equity market and Gold



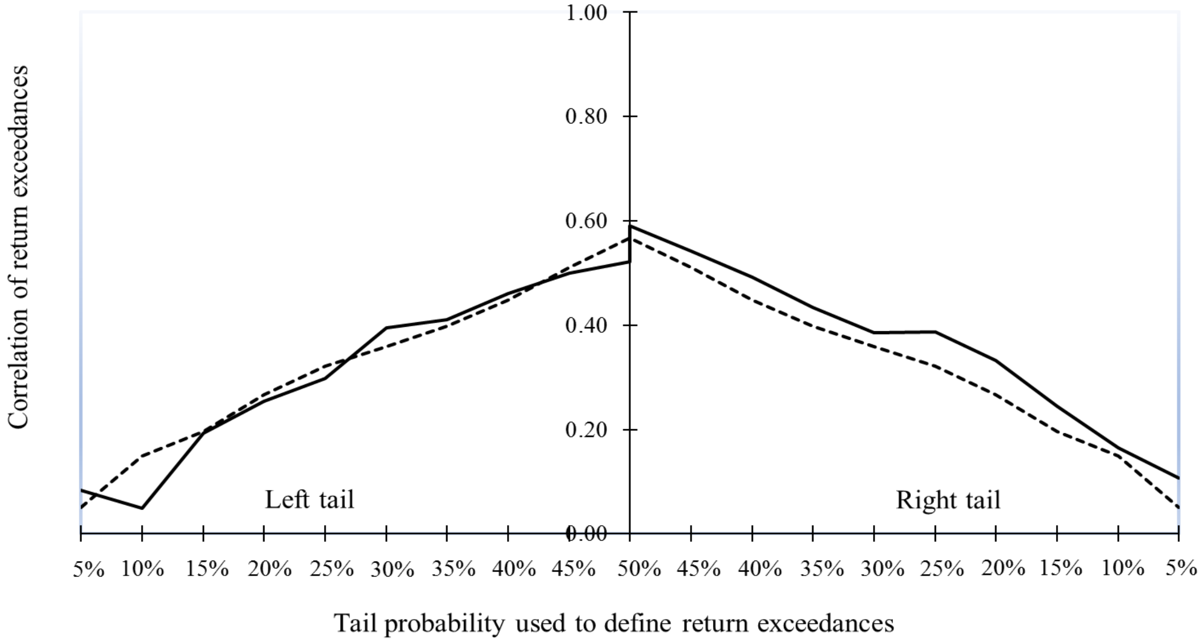
Note: this figure represents the correlation of return exceedances between the European equity markets represented by the STOXX Europe 600 index and Gold. The *solid line* represents the correlation between actual return exceedances obtained from the estimation of the bivariate distribution modeled with the logistic function (see the estimation results in Table 3A). The *dotted line* represents the theoretical correlation between simulated normal return exceedances assuming a bivariate normal distribution with parameters equal to the empirically-observed means and covariance matrix of returns. The value of the tail probability p used to define the threshold for return exceedances ranges from 5% to 50% for both negative return exceedances (left tail) and positive return exceedances (right tail). For a given estimation, the same value of p is taken for both variables: $p = p^{EU} = p^{Gold}$.

Figure 3B. Correlation between return exceedances for the US equity market and Gold



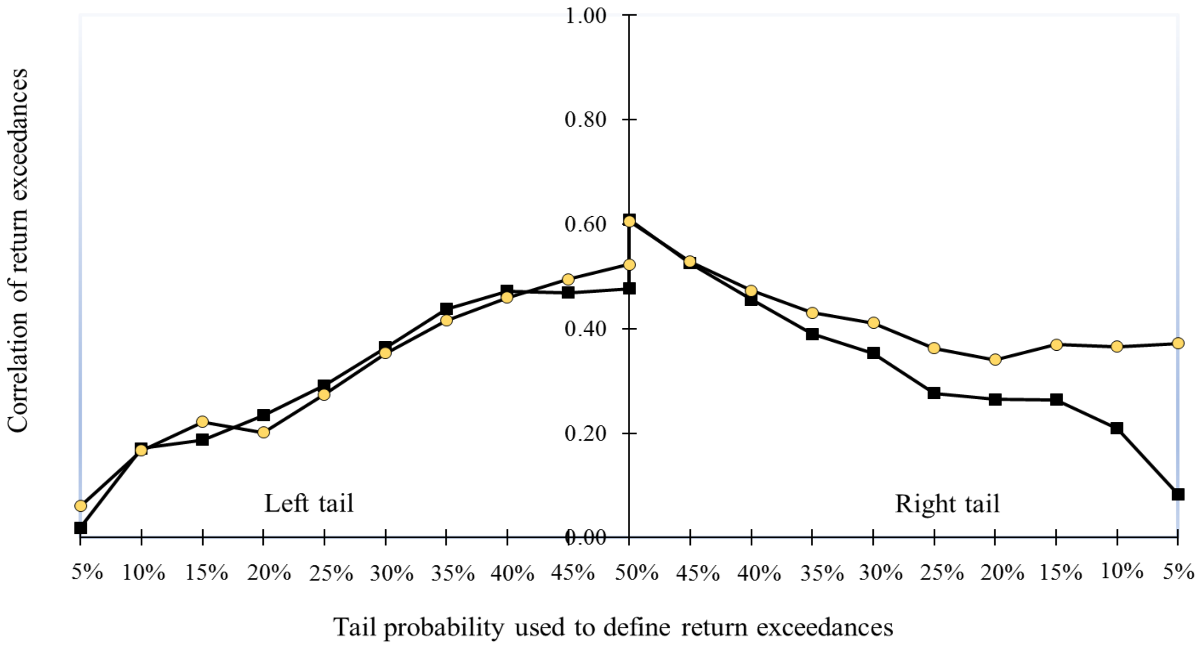
Note: this figure represents the correlation of return exceedances between the US equity markets represented by the S&P 500 index and Gold. The *solid line* represents the correlation between actual return exceedances obtained from the estimation of the bivariate distribution modeled with the logistic function (see the estimation results in Table 3B). The *dotted line* represents the theoretical correlation between simulated normal return exceedances assuming a bivariate normal distribution with parameters equal to the empirically-observed means and covariance matrix of returns. The value of the tail probability p used to define the threshold for return exceedances ranges from 5% to 50% for both negative return exceedances (left tail) and positive return exceedances (right tail). For a given estimation, the same value of p is taken for both variables: $p = p^{US} = p^{Gold}$.

Figure 4. Correlation between return exceedances for Bitcoin and Gold



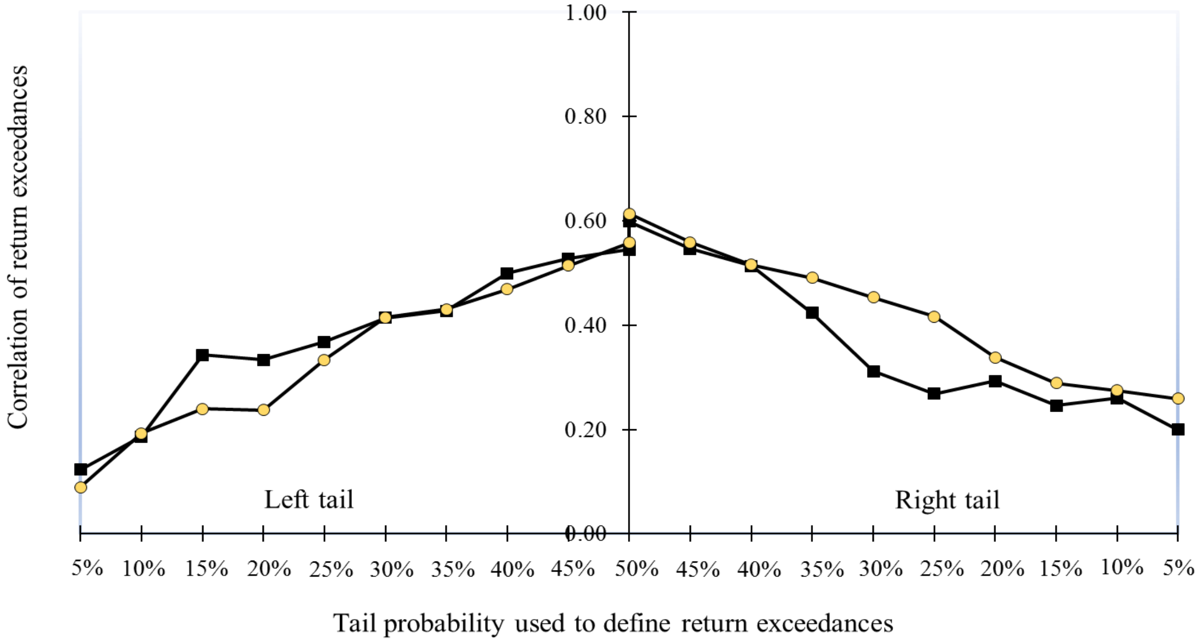
Note: this figure represents the correlation of return exceedances between Bitcoin and Gold. The *solid line* represents the correlation between actual return exceedances obtained from the estimation of the bivariate distribution modeled with the logistic function (see the estimation results in Table 4). The *dotted line* represents the theoretical correlation between simulated normal return exceedances assuming a bivariate normal distribution with parameters equal to the empirically-observed means and covariance matrix of returns. The value of the tail probability p used to define the threshold for return exceedances ranges from 5% to 50% for both negative return exceedances (left tail) and positive return exceedances (right tail). For a given estimation, the same value of p is taken for both variables: $p = p^{BTC} = p^{Gold}$.

Figure 5A. Correlation between return exceedances for the European equity market, Bitcoin or Gold



Note: this figure represents the correlation of return exceedances for the European equity market including Bitcoin or Gold (see the estimation results in Table 5 – Panel A). The *squared points line* represents the correlation between return exceedances for the European equity market and Bitcoin. The *circle points line* represents the correlation between return exceedances for the European equity market and Gold. The value of the tail probability p used to define return exceedances ranges from 5% to 50% for both negative return exceedances (left tail) and positive return exceedances (right tail). For a given estimation, the same value of p is taken for three variables: $p = p^{EU} = p^{BTC} = p^{Gold}$.

Figure 5B. Correlation between return exceedances for the US equity market, Bitcoin or Gold



Note: this figure represents the correlation of return exceedances for the US equity market including Bitcoin or Gold (see the estimation results in Table 5 – Panel B). The *squared points line* represents the correlation between return exceedances for the US equity market and Bitcoin. The *circle points line* represents the correlation between return exceedances for the US equity market and Gold. The value of the tail probability p used to define return exceedances ranges from 5% to 50% for both negative return exceedances (left tail) and positive return exceedances (right tail). For a given estimation, the same value of p is taken for three variables: $p = p^{US} = p^{BTC} = p^{Gold}$.